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LIPPINCOTT'S PRACTICAL PRIMARY ARITHMETIC

SECOND BOOK

BHW

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LIPPINCOTT'S PRACTICAL PRIMARY ARITHMETIC

BEING THE DEVELOPMENT OF THE EARLY IDEAS OF NUMBER AND ITS PRACTICAL APPLICATION IN THE THIRD AND FOURTH GRADES

SECOND BOOK

BY

T. C. BRUFF, C. H. HAYDEN, L. E. WATKINS
OF THE BALTIMORE PUBLIC SCHOOLS

BHW

PHILADELPHIA
J. B. LIPPINCOTT COMPANY

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May 8, 1930

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Electrotyped and Printed by J. B. Lippincott Company
The Washington Square Press, Philadelphia, U. S. A.

PREFACE*

By retaining the best of the old and by combining it with the progressive, the authors of this ARITHMETIC aim to unify and correlate the work in the primary grades, so that there will be no side-stepping or marking time during this period.

The arrangement of the content is by grades from the first to the fourth year inclusive. Each grade is divided into half-year sections of five months. Advancement may be made from section to section, or promotion from grade to grade at stated intervals, or whenever the class has completed the prescribed work.

The first-grade *unit* is presented as an outline or manual for teachers. It consists of approved methods, type lessons, concrete illustrations, and suggestions for the early development of number concepts from 1 to 20. While the experienced and trained teacher may have her own, and possibly more satisfactory methods, yet for the sake of uniformity the general plan of development in this book should be followed by all. For the untrained and unskilled teacher the value of such lessons is incalculable. The greater part of the work in the first and second grades should be oral; children should be encouraged in the free use of materials for seat-work, as impressions may be made through the hand as well as through the eye or the ear.

---

* Written for First and Second Books.

The objective method in introducing new topics is continued throughout the grades. The concrete, however, is dropped as soon as the need for it ceases. The "45 combinations" and other tables are fixed by constant drill, until they become automatic. The idea of ratio begins naturally with the recognition of the relations of familiar objects to similar objects; and these relations, being comprehended by means of the senses, it is a short step to the fractional forms. The familiar terms of the tables of common measurement that come within the daily experience of the child are used from the beginning, but the Tables as a whole are not to be memorized until the fourth year.

Simple problems, adapted to the child's environment, are introduced early. They apply the abstract numbers already learned, and the answers are found by one operation. The oral precedes the written and is intended to stimulate interest, and also through frequent drills to produce quick thinking. Symbols are introduced and explained as they are needed, but no definitions are required in the first three grades.

The authors have attempted throughout the book to give due consideration to the two aspects of arithmetic; to give to each its proper proportion of emphasis, and in so doing to relate one well to the other. The aim has been first *the Why* and then *the How* of the processes; to have the pupil connect what he learns with the vital things of life, and then to acquire skill in what he does by all the means at his command. We hope that the book will prove helpful to many who strive for the *utility* and the *culture* of Arithmetic.

THE AUTHORS.

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PRACTICAL
PRIMARY ARITHMETIC

THIRD GRADE **FIRST HALF YEAR**

LESSON I

Oral:

Review

1 1. $$\left.\begin{array}{r}0\\5\\10\\1\\6\\11\\2\\7\\12\\3\\8\\4\\9\end{array}\right| \text{multiplied by} \left|\begin{array}{l}1\\2\\3\\4\\5\\6\end{array}\right. = ?$$

2. $$\left.\begin{array}{r}2\\5\\3\\6\\9\\4\\7\\8\end{array}\right| \times \left|\begin{array}{l}8\\7\\4\\9\\6\\3\\5\\2\end{array}\right. = ?$$

3. $$\left.\begin{array}{r}2\times10\\10\times\ 2\\3\times10\\10\times\ 3\\4\times10\\10\times\ 4\\5\times10\\10\times\ 5\\6\times10\\10\times\ 6\\7\times10\\10\times\ 7\\8\times10\\10\times\ 8\\9\times10\\10\times\ 9\\10\times10\end{array}\right| = ?$$

4. $$10\times\left|\begin{array}{l}1\\2\\3\\4\\5\\6\\7\\8\\9\\10\end{array}\right. = ?$$

5. $$\left.\begin{array}{r}2\times11\\11\times\ 2\\3\times11\\11\times\ 3\\4\times11\\11\times\ 4\\5\times11\\11\times\ 5\\6\times11\\11\times\ 6\\7\times11\\11\times\ 7\\8\times11\\11\times\ 8\\9\times11\\11\times\ 9\end{array}\right| = ?$$

6. $$11\times\left|\begin{array}{l}1\\2\\3\\4\\5\\6\\7\\8\\9\end{array}\right. = ?$$

7.
$$\left.\begin{array}{r}1\times12\\12\times\ 1\\2\times12\\12\times\ 2\\3\times12\\12\times\ 3\\4\times12\\12\times\ 4\\5\times12\\12\times\ 5\\6\times12\\12\times\ 6\end{array}\right|=?$$

8.
$$12\times\left|\begin{array}{l}1\\2\\3\\4\\5\\6\end{array}\right.=?$$

9.*
$$\left.\begin{array}{r}7\times12\\8\times12\\9\times12\\5\times15\\4\times15\\3\times15\\6\times15\\7\times15\\8\times15\\9\times15\end{array}\right|=?$$

10. Beginning at 5 count by 3's to 100. By 4's, 5's, 6's, 7's, 8's, 9's.

Beginning at 100 count back by 9's, 8's, 7's, 6's, 5's, 4's, 3's.

Beginning at 1, count by 10's to 100; at 2; at 3; at 4; at 5; at 6; at 7; at 8; at 9.

Beginning at 100 count back by 10's; at 99; at 98; at 97; at 96; at 95; at 94; at 93; at 92; at 91.

---

* This work will involve the process of "carrying" done mentally.

11.

16
18
14
12
9 from 20 = ?
19
17
15
13

12.

24
22
20
18
16
14
8 from 12 = ?
11
13
15
17
19
21
23

13.

10
12
14
16
18
20
7 from 22 = ?
24
11
13
15
17
19

14.

10
15
20
11
6
21
12
6 from 17 = ?
22
13
18
23
14
19
24

15.

20
24
23
19
22 5
18 4
21 − 3 = ?
17 2
16 1
12
15
13
14

16.

91 9
81 8
71 7
61 6
51 − 5 = ?
41 4
31 3
21 2
11 1

17.

$$\frac{1}{2} \text{ of } \left| \begin{array}{r} 24 \\ 16 \\ 22 \\ 14 \\ 20 \\ 12 \\ 18 \\ 10 \\ 8 \\ 2 \\ 4 \\ 6 \\ 0 \end{array} \right. = ?$$

18.

$$\frac{1}{3} \text{ of } \left| \begin{array}{r} 24 \\ 3 \\ 21 \\ 6 \\ 18 \\ 9 \\ 15 \\ 12 \\ 0 \\ 27 \\ 36 \\ 30 \\ 33 \end{array} \right. = ?$$

19.

$$\frac{1}{4} \text{ of } \left| \begin{array}{r} 24 \\ 0 \\ 4 \\ 20 \\ 8 \\ 16 \\ 12 \\ 48 \\ 28 \\ 44 \\ 32 \\ 40 \\ 36 \end{array} \right. = ?$$

20.

$$\frac{1}{5} \text{ of } \left| \begin{array}{r} 60 \\ 0 \\ 55 \\ 50 \\ 10 \\ 45 \\ 15 \\ 40 \\ 20 \\ 35 \\ 25 \\ 30 \end{array} \right. = ?$$

21.

$$\frac{1}{6} \text{ of } \left| \begin{array}{r} 72 \\ 36 \\ 66 \\ 30 \\ 60 \\ 12 \\ 54 \\ 18 \\ 48 \\ 24 \\ 42 \\ 6 \\ 0 \end{array} \right. = ?$$

Use this form:

22.

$$\left. \begin{array}{r} 24 \\ 16 \\ 22 \\ \text{etc.} \end{array} \right| \div 2 = ?$$

23.

$$\left. \begin{array}{r} 24 \\ 3 \\ 21 \\ 6 \\ 18 \\ \text{etc.} \end{array} \right| \div 3 = ?$$

also this:

24.

$$\begin{array}{r} 2\underline{)24} \\ ? \end{array} \qquad \begin{array}{r} 3\underline{)24} \\ ? \end{array}$$

$$\begin{array}{r} 2\underline{)16} \\ ? \end{array} \qquad \begin{array}{r} 3\underline{)3} \\ ? \end{array}$$

25. How many weeks in
- 28 days?
- 21 "
- 35 "
- 14 "
- 42 "
- 49 "
- 70 "
- 63 "
- 56 "

26. How many years in
- 72 months?
- 12 "
- 36 "
- 60 "
- 24 "
- 48 "

27. How many days in
- June?
- December?
- August?
- April?
- February?
- November?
- July?
- October?
- January?
- March?
- September?
- May?

28. How many gallons in
- 16 quarts?
- 20 pints?
- 48 gills?
- 20 quarts?
- 40 pints?

29. How many quarts in
- 7 gallons?
- 28 pints?
- 40 gills?
- 3 gallons?
- 40 pints?
- 16 gills?

30. How many pints in
- 16 gills?
- 16 quarts?
- 3 gallons?
- 10 gallons?
- 30 quarts?
- 48 gills?

31. How many inches in
- 10 feet?
- 7 feet?
- 6 feet?
- 4 feet?
- 5 feet?
- 2 yards?
- 1 yard?

32. How many feet in
36 inches?
48 inches?
60 inches?
24 inches?
3 yards?
6 yards?
8 yards?

33. When is
Christmas?
Independence Day?
Decoration Day?
Washington's Birthday?
Your birthday?
New Year's Day?
Memorial Day?
Columbus Day?
Flag Day?

34. How many yards in
36 inches?
9 feet?
12 feet?
18 feet?
15 feet?

2 Written:

1.
614
+219
817
619
493

2.
917
+817
634
179
328

3.
649
+276
714
209
763

4.
914
+316
437
217
917

5.
219
+176
154
249
176

6.
179
+213
174
914
187

7.
198
+674
198
619
718

8.
914
+419
149
941
194

9. Write in words each addend and each sum. (Always prove.)

10. $\begin{array}{r}917\\-619\\\hline\end{array}$ **11.** $\begin{array}{r}2176\\-193\\\hline\end{array}$ **12.** $\begin{array}{r}1700\\-219\\\hline\end{array}$ **13.** $\begin{array}{r}817\\-148\\\hline\end{array}$ **14.** $\begin{array}{r}1649\\-891\\\hline\end{array}$

15. $\begin{array}{r}1000\\-216\\\hline\end{array}$ **16.** $\begin{array}{r}8176\\-2981\\\hline\end{array}$ **17.** $\begin{array}{r}1761\\-209\\\hline\end{array}$ **18.** $\begin{array}{r}2179\\-1782\\\hline\end{array}$

19. $\begin{array}{r}193\\\times 6\\\hline\end{array}$ **20.** $\begin{array}{r}438\\\times 9\\\hline\end{array}$ **21.** $\begin{array}{r}435\\\times 7\\\hline\end{array}$ **22.** $\begin{array}{r}1248\\\times 5\\\hline\end{array}$ **23.** $\begin{array}{r}219\\\times 8\\\hline\end{array}$

24. $\begin{array}{r}916\\\times 6\\\hline\end{array}$ **25.** $\begin{array}{r}1419\\\times 7\\\hline\end{array}$ **26.** $\begin{array}{r}216\\\times 9\\\hline\end{array}$ **27.** $\begin{array}{r}7159\\\times 4\\\hline\end{array}$

28.
What remains when you divide
14
29
36
17
40
37
39
29
25
19
18
17
20
by
4
2
6
8
9
7
5
3

29. How many
2's
3's
4's
5's
6's
7's
8's
9's
and what besides make
29
27
25
23
21
19
17
15
28
26
24
22
20
18

3 What does

XII
IX
XI
VIII
XX
XVIII
XIII
XIX
XXIV
XXX
L
XL

equal?

Drill:

Write in Roman numerals the numbers from 1 to 50.

Write the even numbers up to 100.

Write the odd numbers up to 100.

Write the multiplication tables, 2 to 9.

Write all combinations of numbers, 2 to 18.

How many units in one ten? tens in one hundred? hundreds in one thousand?

How many tens in 19 units; 29 units; 39 units, etc.; hundreds in 16 tens; 26 tens; 36 tens, etc.; thousands in 18 hundreds; 28 hundreds, etc.?

LESSON II

Multiplication Tables Completed

4 8×1
8×2
8×3
8×4
8×5
8×6
8×7
8×8
8×9
$8 \times 10 = 80$
$8 \times 11 = 88$
$8 \times 12 = 96$

Say the *8 table* as far as 8×9. There are a few more products to find before the table will be complete. You must find how much 8×10, 8×11, 8×12 are. You can find these answers two ways,—by adding and by multiplying. Find them both ways.

$$\begin{array}{r}10\\ \times 8\\ \hline 80\end{array}\qquad\qquad\begin{array}{r}11\\ \times 8\\ \hline 88\end{array}\qquad\qquad\begin{array}{r}12\\ \times 8\\ \hline 96\end{array}$$

Study these new answers now with the others so that you will know them all well and QUICKLY.

9×1
9×2
9×3
9×4
9×5
9×6
9×7
9×8
$9 \times 9 = 81$
$9 \times 10 = 90$
$9 \times 11 = 99$
$9 \times 12 = 108$

Say the *9 table* as far as 9×8. Now by adding find how much 9×9 are. The three other answers you can obtain as before in two ways. Find them both ways; one will check the other.

$$\begin{array}{r}10\\ \times 9\\ \hline 90\end{array}\qquad\qquad\begin{array}{r}11\\ \times 9\\ \hline 99\end{array}\qquad\qquad\begin{array}{r}12\\ \times 9\\ \hline 108\end{array}$$

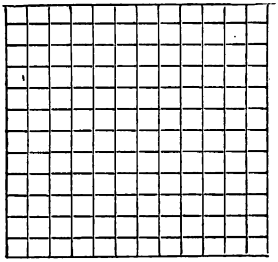
10×1
10×2
10×3
10×4
10×5
10×6
10×7
10×8
10×9
10×10
10×11
10×12

11×1
11×2
11×3
11×4
11×5
11×6
11×7
11×8
11×9
11×10
11×11
11×12

NOTE.—To find the answers to the *10* and *11 tables* will make excellent exercises for seat-work—using both the addition and multiplication methods "to make sure." Have the pupils see that multiplying by *10* simply annexes a *0* to the multiplicand; then give added practice by multiplying larger numbers by *10,* as 10×25; 10×36; etc.

The Square Foot

5 One foot or 12 inches



(Outline a square foot on the blackboard.) How long is each side of this square? We call such a square a *square foot.* Divide each side into inches; then draw lines across. Each small division will be a square too. What will be the length of its sides? We call such a square a *square inch.*

12 × 1 = 12
12 × 2 = 24
12 × 3 = 36
12 × 4 = 48
12 × 5 = 60
12 × 6 = 72
12 × 7 = 84
12 × 8 = 96
12 × 9 = 108
12 × 10 = 120
12 × 11 = 132
12 × 12 = 144

How many square inches are on one row? on two rows? on three rows? etc. Find the answers by adding 12 and 12, then 12 to the sum; and so on till the *12 table* is complete. Comparison with results in the tables already learned will check these new results, and 12×12 can be obtained two ways.

$$\begin{array}{r} 12 \\ \times 12 \\ \hline 144 \end{array}$$

(Now by much drill in oral and in written form have these twelve tables memorized; and by frequent repetition and use strive to have them become automatic exercises.)

6 Oral:

How many squares in one row of the square foot? in the 12 rows? How many square inches in the square foot? How many square inches in 6 rows? What part of a square foot is that? How many square inches in $\frac{1}{2}$ sq. ft.? How many square inches in 3 rows? What part of a sq. ft. is that? How many square inches in $\frac{1}{4}$ of a square foot? How many square inches in 4 rows? What part of a square foot is that? How many square inches in $\frac{1}{3}$ of a sq. ft.? 1 sq. ft. = how many sq. in.? $\frac{1}{3}$ of a sq. ft. = how many sq. in.? $\frac{1}{2}$ of a sq. ft. = how many sq. in.? $\frac{1}{4}$ of a sq. ft. = how many sq. in.? How many inches on the edges of a square foot? How many square inches touch the edges of a square foot?

7 Written exercises:

1. How many sq. in. in 2 sq. ft.?; in 3, 4, 5, 6, 7, 8, 9, 10 sq. ft.?
2. How much more are 190 sq. in. than 1 sq. ft.?
3. How much less are 258 sq. in. than 3 sq. ft.?
4. How many sq. in. in 1 sq. ft.? in $\frac{1}{2}$ sq. ft.? $\frac{1}{3}$ sq. ft.? and $\frac{1}{4}$ sq. ft.?
5. If an oblong is two feet long, and one foot wide, how many square feet would it contain? How many square inches?

6. $\begin{array}{r} 926 \\ \times 9 \\ \hline \end{array}$ 7. $\begin{array}{r} 254 \\ \times 8 \\ \hline \end{array}$ 8. $\begin{array}{r} 198 \\ \times 7 \\ \hline \end{array}$ 9. $\begin{array}{r} 1268 \\ \times 6 \\ \hline \end{array}$ 10. $\begin{array}{r} 986 \\ \times 9 \\ \hline \end{array}$ 11. $\begin{array}{r} 468 \\ \times 9 \\ \hline \end{array}$

12. $\begin{array}{r} 619 \\ \times 12 \\ \hline \end{array}$ 13. $\begin{array}{r} 486 \\ \times 12 \\ \hline \end{array}$ 14. $\begin{array}{r} 864 \\ \times 11 \\ \hline \end{array}$ 15. $\begin{array}{r} 267 \\ \times 11 \\ \hline \end{array}$ 16. $\begin{array}{r} 269 \\ \times 11 \\ \hline \end{array}$ 17. $\begin{array}{r} 715 \\ \times 9 \\ \hline \end{array}$

18. What would a dozen yards of silk cost, if one yard cost 65 cents?
19. What will 12 yards of ribbon cost, if 1 yard costs $0.19?

Multiply:

20. 617 by 9
21. 1976 by 11
22. 4163 by 11
23. 694 by 7
24. 789 by 8
25. 986 by 9
26. 4163 by 12
27. 1949 by 8
28. 927 by 12
29. 287 by 8
30. 4163 by 9
31. 9876 by 9

32. Write the name of each of the above products.

33. From 400 take 9×12; 8×12; 12×12; 7×9; 8×9; 9×9.

34. Add the above six remainders.

LESSON III

8

Multiplication

Operation

$$\begin{array}{rll} 39 & \cancel{9} & \cancel{1}\cancel{8} \\ \times 21 & \cancel{3} & 7 \\ \hline 39 & & \\ 78 & & \\ \hline 819 = \text{product} & & \end{array}$$

Proof-check

$$\begin{array}{rll} 21 & \cancel{9} & \cancel{3} \\ \times 39 & \cancel{1}\cancel{8} & \cancel{6} \\ \hline 189 & & \\ 63 & & \\ \hline 819 = \text{product} & & \end{array}$$

Operation

$$\begin{array}{r} 48 \\ \times 40 \\ \hline 1920 = \text{product} \end{array}$$

Proof-check

$$\begin{array}{rll} 40 & 0 & 0 \\ \times 48 & 32 & 16 \\ \hline 320 & & \\ 160 & & \\ \hline 1920 = \text{product} & & \end{array}$$

We read this 39 multiplied by 21; and it means that 39 is to be taken 21 times. We call 39, the number to be multiplied, the *Multiplicand;* and 21, the number by which 39 is to be multiplied, the *Multiplier*. The result of a multiplication, as we know, is called the *Product*.

Now we are multiplying by 21, a number of 2 figures, 1 in the *units* place, 2 in the *tens* place. We multiply 39 by the 1 unit and begin to write the result in the units place. We then multiply 39 by the 2 tens and begin at the tens place to write the result. We add these two partial products to get the whole product. We can check this result by reversing the multiplication, and multiply 21 by 39. When the multiplier has *0* in the units place, we write *0* in the product in the units place; then we multiply by the figure in the tens place, writing the result in the tens place, as before.

Multiply:

1. 24 by 16 2. 75 by 23 3. 67 by 55 4. 56 by 15
5. 38 by 18 6. 87 by 32 7. 58 by 46 8. 94 by 36
9. 46 by 19 10. 93 by 45 11. 92 by 22 12. 79 by 42
13. 53 by 17 14. 81 by 56 15. 89 by 33 16. 35 by 16
17. 62 by 15 18. 78 by 44 19. 74 by 51 20. 28 by 75

21.	22.	23.	24.	25.	26.
$2.75	$3.20	$0.85	$1.50	$0.38	$2.25
×15	×42	×36	×75	×27	×50

(Remember to use a *point* in the product to separate cents from dollars.)

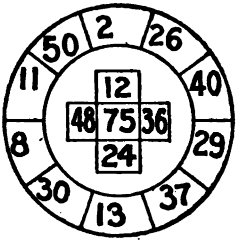
Multiply: 27. 234 by 13; by 25; by 30; by 42; by 51
28. 105 by 14; by 23; by 47; by 65; by 89
29. 381 by 72; by 68; by 54; by 32; by 40
30. 408 by 16; by 23; by 44; by 62; by 50
31. 693 by 22; by 54; by 18; by 71; by 20

32.	18	25	37	62	76	85	22	31
	×2	×3	× 4	× 5	×6	×7	×8	×9
33.	18	25	37	62	76	85	22	31
	×20	×30	×40	×50	×60	×70	×80	×90

34. Compare the products obtained in the upper line with those in the lower. Now see if you can give these products at once.

	9		8		3		2		6
	6		7		6		5		7
	5		3		5		8		1
20 ×	4 = ?	30 ×	4 = ?	40 ×	4 = ?	50 ×	9 = ?	60 ×	2 = ?
	3		1		2		6		3
	2		6		1		3		5
	7		2		8		7		9
	8		9		9		4		4
					7				8

35. Written exercise:



Multiply each number in the centre by all the numbers in the ring.

9

Problems

(1) There are 16 ounces in 1 lb. How many ounces are in 25 lbs.?

(2) There are 36 inches in 1 yard. How many inches are in 34 yards?

(3) There are 24 hours in 1 day. How many hours are in February?

(4) If 1 hogshead holds 63 gallons, how many gallons will 22 hogsheads hold?

(5) If 1 cheese weighs 25 pounds, how many pounds do 46 cheeses weigh?

(6) If 1 tub of butter weighs 32 pounds, how many pounds do 18 tubs weigh?

(7) What will 58 pounds of butter cost at $0.36 a pound?

(8) What will 34 yards of cloth cost at $1.75 a yard?

(9) What will 89 acres of land cost at $250 an acre?

(10) Each chalk box contains 144 pieces. How many pieces are in 2 dozen boxes?

(11) Each box of chalk costs $0.15. What will 3 dozen boxes cost?

(12) Each box lasts our class 36 days. How many days will 13 boxes last?

(13) What will 18 bags of grain weigh, if each weighs 35 pounds?

(14) What will 20 barrels of flour weigh, if each weighs 196 pounds?

(15) What will 35 sacks of flour weigh, if each weighs 24 pounds?

(16) If the wages of each workman is $24, how much will be paid to 25 men?

(17) If each man worked 48 hours, how many hours did all work?

(18) If they make 32 feet of road in 1 day, how much will they make in 18 days?

(19) If we sell 45 baskets of peaches a day, how many baskets shall we sell in 14 days?

(20) If the freight is $0.15 a mile, what will it be for 90 miles?

LESSON IV

10

Numbers of 3 Orders

Millions	Thousands			Units		
Millions	100-Thousands	10-Thousands	Thousands	Hundreds	Tens	Units
						1
					1	0
				1	0	0
			1	0	0	0
		1	0	0	0	0
	1	0	0	0	0	0
1	0	0	0	0	0	0

We have already learned that 10 units = 1 ten; that 10 tens = 1 hundred; that 10 hundreds = 1 thousand. Now we shall learn that 10 thousands contain one more figure than 1 thousand, and occupy one more place to the left; that 100 thousands contain two more figures than 1 thousand, and occupy two more places to the left. These 3 orders (the 4th, 5th, and 6th, from the right) make the second or thousands period, and are separated by a comma from the first or units period; thus, 325,487. The 7th place from the right is called the millions order, and is separated by a comma from the thousands period; so 1 in the 7th order is read 1 million. You must memorize the orders, beginning at the right.

Units—tens—hundreds.
Thousands—ten-thousands—hundred-thousands.
Millions.

1. Read these numbers: Do not use *and* between the orders.

0, 000, 000
76, 485
87, 038
195, 203
346, 005
203, 489
1, 675, 000
500, 500
2, 030, 594
3, 098, 605
4, 960, 530

0, 000, 000
689
84, 035
20, 690
300, 462
489, 754
836, 000
1, 000, 000
1, 800, 000
1, 890, 000
1, 895, 500

0, 000, 000
1, 056, 056
890, 890
625, 025
86, 086
91, 010
320, 564
408, 867
1, 111, 111
101, 101
89, 089

2. Write the following numbers, being careful to place each figure in the proper place, and figures of the same order exactly under each other. Use commas between the periods.
3. Five thousand, eighty-four.
4. Twenty-five thousand, four hundred fifty.
5. Two hundred twenty thousand, three hundred five.
6. One million, thirty thousand, forty-six.
7. Six hundred thousand, six hundred six.
8. Fifty-five thousand, three hundred thirty.
9. Two million, eight hundred three thousand, five hundred sixty-one.

10. Fifty-three thousand, fifty-three.
11. Eight thousand, nine hundred seventy-five.
12. One million, one thousand, one.
13. One million, one hundred thousand, one hundred.
14. One hundred fifty-six thousand, one hundred fifty-six.
15. One hundred eleven thousand, eleven.
16. Five hundred thousand, five hundred five.
17. Five thousand, two hundred eighty.
18. One million, two hundred fifty thousand, five.
19. Nine hundred ninety-nine thousand, nine hundred ninety-nine.
20. Eighty-seven thousand, nine hundred thirty-six.
21. Two million, five hundred six thousand, seven hundred four.
22. Three million, fifty-four thousand, six hundred thirty-one.

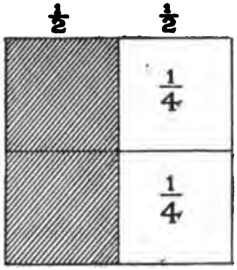
11 Write, first in figures, then in words:

1. The number of this year.
2. The year of your birth.
3. The day of the month.
4. The last numbered page in your book.
5. The number of days in a year. (See teacher's calendar.)
6. The number of days that are gone. (See teacher's calendar.)

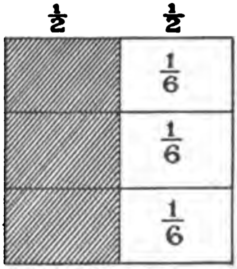
7. The number of days left in the year. (See teacher's calendar.)
8. The number of days in this month. (See teacher's calendar.)
9. The number of years of your age.
10. The year of the Declaration of Independence.
11. The day of the month that marks Washington's birthday.
12. The largest number you can read and write.

LESSON V

12 One-Half to Equivalent Fractions

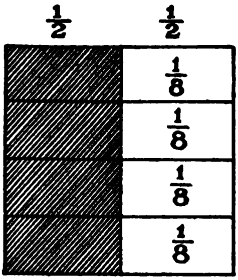


1. (Draw the vertical dividing line.) How have I divided this oblong? (2 equal parts.) Each part is what part of the whole oblong? How many halves are in the oblong? (Draw the horizontal dividing line.) I have now divided each half into two equal parts. How many small parts are there all of the same size? What part of the whole oblong is each of the small parts? Each half equals how many fourths? Then $\frac{1}{2}$ of a thing = $\frac{2}{4}$ of the thing. Make diagrams showing this, using different shaped oblongs, squares, circles.

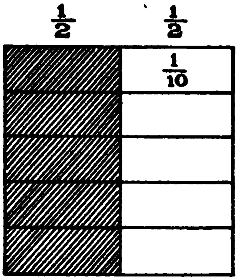


2. Here I divide the oblong into two equal parts. (With the vertical line.) What is the name of each? Then I divide each half into three equal parts. (With the horizontal lines.) How many of the small parts in the whole oblong? Each one is what part of the whole? Each half equals how many sixths? $\frac{1}{2}$ of a thing = $\frac{3}{6}$ of the thing.

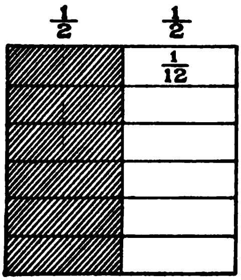
Show that this is true in other cases by using different shaped diagrams.



3. Again I divide the whole oblong into ? halves? If I divide each half into 4 equal parts, how many such parts are in the whole oblong? Each small part is what part fraction of the whole oblong? Each half equals how many small parts? $\frac{1}{2}$ of a thing = $\frac{4}{8}$ of the thing.



4. And so on, by diagrams show that $\frac{1}{2} = \frac{5}{10}$; $\frac{6}{12}$; $\frac{8}{16}$; $\frac{10}{20}$.



5. Seat-work: Let the pupils have considerable practice in illustrating these equivalents with various sized and shaped figures. Colored crayons or water-colors will aid much in making the work effective in appearance, and thereby impress its purpose upon them.

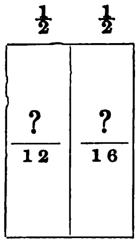
13 Oral exercises:

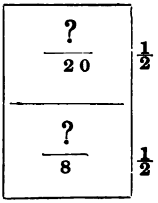
1.
$$\frac{1}{2} = \left|\begin{array}{l}\frac{?}{4}\\[4pt] \frac{?}{6}\\[4pt] \frac{?}{8}\\[4pt] \frac{?}{10}\\[4pt] \frac{?}{12}\\[4pt] \frac{?}{16}\\[4pt] \frac{?}{20}\end{array}\right.$$

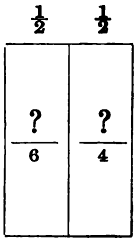
2. How many fourths
sixths
eighths
tenths
twelfths
sixteenths
twentieths $= \frac{1}{2}$?

3. What is $\frac{1}{2}$ of 24 inches?
$\frac{6}{12}$ of 18 inches?
$\frac{4}{8}$ of 50 inches?
$\frac{5}{10}$ of 100 cents?
$\frac{8}{16}$ of 80 apples?
$\frac{3}{6}$ of 60 boys?
$\frac{10}{20}$ of 22 eggs?

4. What part of $\frac{1}{2}$ is $\frac{1}{4}$? is $\frac{1}{6}$? is $\frac{1}{8}$? is $\frac{1}{10}$? is $\frac{1}{12}$? is $\frac{1}{16}$?







5. Which is greatest, $\frac{1}{2}$ of 10, $\frac{3}{6}$ of 8, or $\frac{5}{10}$ of 2?
6. How much more is $\frac{4}{8}$ of 30 than $\frac{2}{4}$ of 24?
7. Take $\frac{10}{20}$ of 100 from $\frac{1}{2}$ of 120.

8. $6 = \frac{1}{2}$ of what?
9. $20 = \frac{8}{16}$ of what?
10. $30 = \frac{8}{16}$ of what?
11. $10 = \frac{10}{20}$ of what?
12. $18 = \frac{3}{6}$ of what?
13. $50 = \frac{3}{6}$ of what?
14. $13 = \frac{2}{4}$ of what?
15. $24 = \frac{5}{10}$ of what?
16. $100 = \frac{6}{12}$ of what?

14 Written exercises:

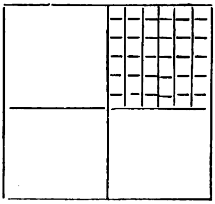
1. Add 268 square inches, 916 squares inches, 87 square inches, 9 square inches, 958 square inches.
2. How many square inches in 9 square feet?
3. Take the answer of the second problem from the answer of the first.
4. My desk top contains 6 square feet. How many square inches is that?
5. One half of it is covered with paper; how many square inches are covered?

1. How much less is one square foot than 200 square inches?
2. How many square inches in 8 square feet? 9 square feet? 12 square feet? 7 square feet?
3. Add the last four sums.
4. $9 \times 698 =$ $8 \times 189 =$ $9 \times 632 =$ $\begin{array}{r}1297\\ \times 8\\ \hline\end{array}$ $\begin{array}{r}9863\\ \times 8\\ \hline\end{array}$ $\begin{array}{r}8014\\ \times 12\\ \hline\end{array}$
   $12 \times 432 =$ $9 \times 495 =$ $9 \times 367 =$
5. $12\overline{)144}$ $7\overline{)84}$ $12\overline{)96}$ $9\overline{)81}$ $12\overline{)132}$ $11\overline{)132}$

1. Divide a square foot into square inches. How many?
2. Divide the square foot into 4 equal parts. How many square inches in each?

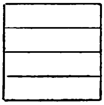
3. How many fourths in one half? How many square inches in two fourths?
4. How many square inches in three fourths of the square foot?
5. How many square inches in four fourths of the square foot?
6. Four fourths of the square foot equal what part of the square foot?

1. Divide a square foot into fourths this way:



How many inches around (call it perimeter) $\frac{1}{4}$ of the square foot? Around $\frac{1}{2}$ of the square foot?

2. How many square inches in $\frac{1}{4}$ of the square foot? In $\frac{1}{2}$ of the square foot? In $\frac{3}{4}$ of the square foot?
3. Divide it this way: How far around each fourth? Around each half? Around $\frac{3}{4}$ of the square foot?



LESSON VI

15

Review and Drill

Written exercises:

1.
$$\text{How much are } \left.\begin{array}{r}2\\20\\3\\30\\4\\40\\5\\50\\6\\60\end{array}\right| \text{ times } \left|\begin{array}{r}9\\0\\10\\1\\11\\2\\12\\3\\8\\4\\7\\5\\6\end{array}\right. ?$$

2.
$$\left.\begin{array}{r}7\\70\\8\\80\\9\\90\\10\\100\\11\\110\\12\\120\end{array}\right| \text{ times } \left|\begin{array}{r}6\\5\\7\\4\\8\\3\\12\\2\\11\\1\\10\\0\\9\end{array}\right. = ?$$

3. 698 multiplied by 9 = what?
4. 2168 multiplied by 8 = what?
5. 876 multiplied by 7 = what?

Write each answer in words. Multiply each answer by 10, and write the new answer in words.

How much are:

6. 9×8964 7. 8×9876 8. 7×9999
9. 8×4208 10. 7×2345 11. 8×7777
12. 7×2198 13. 9×8743 14. 9×6666

Write each answer in words.

Multiply each answer by 10; then write the new answer in words.

16 Oral exercises:

Give instant answers:

1\.
9
19
7
17
3
13
8
18
28
6
16
26
4
24
+9 = ?

2\.
3
33
5
15
25
7
17
27
2
22
42
4
14
8
18
+8 = ?

3. Add 7 to 9, 8, 7, 1, 2, 3, 6, 5, 4, 12, 11, 10.

4. Add 6 to 1, 11, 9, 19, 2, 12, 8, 18, 3, 13, 7, 17, 4, 14, 6, 16, 5, 15.

5. Add 5 to 6, 16, 26, 36, etc.

6. Add 9 to 7, 17, 27, 37, etc.

7. Add 8 to 9, 8, 7, 19, 18, 17, 6, 5, 4, 16, 15, 14.

8. Take 9 from 14, 24, 34, 44, etc.
9. " 8 from 15, 16, 9, 27, 36, 12, 23, 16.
10. " 7 from 9, 8, 11, 12, 14, 13, 20, 15, 16, 7.
11. " 6 from 10, 20, 11, 9, 12, 8, 13, 7, 16, 15, 14.
12. " 5 from 20, 30, 40, etc.; 11, 21, 31, 41, etc.

17 Give quotients and remainders, if any:

1.
63
84
145
16
94
87
49 ÷12
39
100
25
67
79
123
132

2.
17
29
36
49
54
69
87 ÷9
60
21
75
94
100
112

3.
64
72
80
16
24
32 ÷8
48
88
96
40
8
56

4.
54
63
75
46
39
57 ÷7
28
25
79
9
17
84

5.
9
18
23
24
32
39 ÷6
42
48
59
61
69
73

6.
60
40
20
0
55
35
$\frac{1}{5}$ of 15 = ?
50
30
10
45
25
5

7.
12
36
48
16
40
$\frac{1}{4}$ of 20 = ?
44
4
28
8
0
32

8.
$\frac{1}{2}$ = how many
fourths?
sixths?
tenths?
sixteenths?
twentieths?
twelfths?
eighths?
twenty-fourths?

9. $\frac{1}{6}$
$\frac{1}{8}$
$\frac{1}{4}$
$\frac{1}{10}$
$\frac{1}{12}$
$\frac{1}{16}$
$\frac{1}{20}$
= what part of $\frac{1}{2}$?

10. 1 dollar
1 nickel
1 quarter
1 dime
3 nickels
1 half-dollar
4 quarters
6 dimes
2 quarters
7 nickels
3 dollars
7 quarters
= how many cents?

11.
1 gallon
1 quart
8 gills
3 quarts
3 gallons
16 gills
8 quarts
10 quarts
36 gills
½ quart
2 gallons
= how many pints?

12.
1 square foot
2 square feet
½ square foot
⅓ square foot
⅔ square foot
¼ square foot
$\frac{2}{4}$ square foot
¾ square foot
= how many square inches?

13. IX
XXIV
XXX
XIX
XII
XL
= ?

L
XXXV
XXIII
XXII
XLVI
= ?

18 Written exercises:

1.	2.	3.	4.	5.	6.
9176	217	918	9000	7123	6104
+4198	+9864	+9180	−2109	−1298	−3217
7164	329	918			
9874	7168	91	7. 6194	8. 76943	
3219	764	6819	×9	×8	
6194	321	2718			

Multiply: 9. 6197 by 6 10. 7936 by 3 11. 9016
12. 2198 by 4 13. 2168 by 9 ×7

19 Check each problem. Write each result in words.

1. Add 29, 614, 876, 9143, 287, 4398.
2. Add 2076, 20769, 967, 9670, 96702, 627.
3. Take 21768 from fifty thousand, one hundred.
4. From 21870 take 9876.
5. What will 9 times 68943 be?
6. How much greater than 299 square inches is 5 hundred square inches?
7. Write the even numbers up to fifty in Roman numerals.
8. Write the odd numbers up to forty-nine in Roman numerals.

9.

Count by	5's 4's 3's 6's 7's 8's 9's 10's	beginning at	0 1 2 3 4 5 6 7 8 9

10.

Count back from 100 by	2's 3's 4's 9's 8's 7's 5's 6's

11. Multiply 6894 by
4, 40, 9, 8, 7, 12, 10, 6, 11, 5

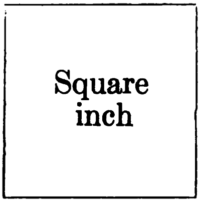
12. Multiply 917 by
4, 40, 9, 3, 30, 8, 12

LESSON VII

Surface Measure

20 (Pupils to be supplied with pieces of paper 1″×1″, 4″×1″, 4″×2″.) Take up your piece of paper like this. How long is it? How wide is it? What is it called? We use the square inch as a measure of surface or *area* (new term).

Using this measure, find how many square inches are in the smaller oblong paper. How long is the oblong? How wide is it?



Use the measure with the larger oblong. How long is it? How many square inches in the bottom row? How wide is it? How many rows of 4 square inches? How many square inches in the oblong? (2×4 square inches = 8 square inches.)

Form to be used:

$$\begin{array}{l} 5 \text{ sq. in. on 1 row} \\ \underline{\times 3} \\ 15 \text{ sq. in. in oblong} \end{array} \quad \text{or} \quad \begin{array}{l} 3 \text{ sq. in. on 1 row} \\ \underline{\times 5} \\ 15 \text{ sq. in. in oblong} \end{array}$$

How many square inches in an oblong:

5 inches by 3 inches?
18 inches by 3 inches?
4 inches by 3 inches?
10 inches by 6 inches?
6 inches by 4 inches?
15 inches by 4 inches?
9 inches by 5 inches?
12 inches by 6 inches?

Measure the top of your book. How many inches long is it? How many inches wide? How many square inches of surface in it?

Measure the paper you use in the same way.

Measure your checker-board at home in the same way.

The square inch is too small a measure for large surfaces like the top of your desk, the blackboard, etc. Let us use the ruler and measure off on the board a surface one foot long, one foot wide. What do we call this? How many inches long is it? How many inches wide? How many square inches in each row? How many rows of 12 square inches? How many square inches in the 12 rows? How many square inches in a square foot?

Learn: 144 square inches = 1 square foot.

1. How many square inches in:
   2 square feet? 3 square feet? 5 square feet?
   6 square feet? 4 square feet? 7 square feet?
   $\frac{1}{2}$ square foot = 6 rows of 12 square inches each,
   $\frac{1}{3}$ square foot = 4 rows of 12 square inches each,
   and so on.
2. How many square inches in: $\frac{1}{2}$ square foot? $\frac{1}{3}$ square foot? $\frac{1}{4}$ square foot? $\frac{1}{6}$ square foot? $\frac{1}{12}$ square foot?
3. Measure the top of your desk. How many feet

long is it? How many feet wide? How many square feet of surface in the top?

4. Measure the top of the table in the same way; the surface of the blackboard; the mat at the door; your dining-table at home.
5. How many square feet in an oblong: 3 feet by 2 feet; 14 feet by 3 feet; 25 feet by 14 feet; 6 feet by 12 feet?
6. 8 feet by 1 foot; 23 feet by 3 feet; 12 feet by 15 feet; 14 feet by 2 feet?

21 Both the square inch and the square foot are too small a measure for the floor, the wall, the garden, the sidewalk. Take the yard stick and measure off on the floor a surface which is one yard long, and one yard wide. We call this one square yard. (1 sq. yd.) How many feet long is this? How many feet wide? How many square feet in 1 row? How many rows? How many square feet in 3 rows? How many square feet in the square yard?

Learn: $\boxed{\text{9 square feet} = \text{1 square yard.}}$

1. How many square feet in:
   2 square yards; 3 square yards; 4 square yards; 6 square yards; 8 square yards?
2. $\frac{1}{3}$ of a square yard; $\frac{2}{3}$ of a square yard?

   How many square yards in the area of:
3. A floor 5 yards long, 3 yards wide?

4. A wall 6 yards long, 5 yards high?
5. A rug 4 yards long, 3 yards wide?
6. A garden 7 yards long, 5 yards wide?
7. A sidewalk 5 yards long, 4 yards wide?
8. A road 18 yards long, 5 yards wide?

Measure the school-room floor. How many yards long? How many yards wide? How many square yards of surface?

Measure a floor in your own home in the same way.

9. 1
3
2
4 | sq. yd. = ? sq. ft.
$\frac{1}{3}$
$\frac{2}{3}$

10. 2
3
1
$\frac{1}{2}$ | sq. ft. = ? sq. in.
$\frac{1}{4}$
$\frac{1}{3}$

11. 9
27
18
36 | square feet = ? square yards
45
63
72

22 Written Problems

(1) Our school paper is 10 inches long, 8 inches wide. How many square inches in a sheet?

(2) Our piano cover is 6 feet long, 4 feet wide. How many square feet in the cover?

(3) The porch of a house is 5 yards long, 3 yards wide. How many square yards of surface?

(4) My handkerchief is 12 inches long, 12 inches wide. How many square inches of material?

(5) The walk in the garden is 10 feet long, 2 feet wide. How many square feet of walk?

(6) The dining-room rug is 4 yards long, 3 yards wide. How many square yards does it cover?

(7) A postal card is 6 inches long, 4 inches wide. How many square inches in its surface?

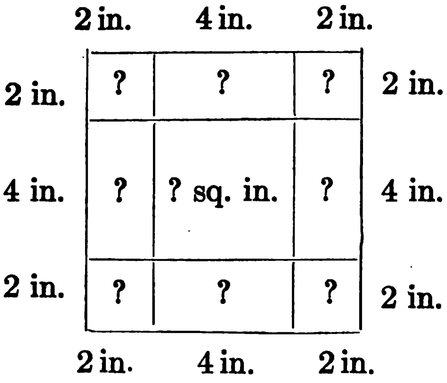
(8) The tennis court is 12 yards long, 8 yards wide. How many square yards in the court?

(9) Our school-room is 24 feet long, 18 feet wide. How many square feet of floor?

(10) Mary's hair ribbon is 30 inches long, 3 inches wide. How many square inches of ribbon are in it?

$\frac{1}{6}$ of	÷ 4	6 ×	5 ×
$\frac{1}{2}$ of	12 sq. in.		÷ 3
$\frac{1}{3}$ of			÷ 6
$\frac{1}{4}$ of	8 ×	9 ×	÷ 2

$\frac{1}{2}$ of	$\frac{1}{8}$ of	2 ×	4 ×
$\frac{1}{4}$ of	24 sq. in.		5 ×
$\frac{1}{6}$ of			10 ×
$\frac{1}{3}$ of	$\frac{1}{12}$ of	3 ×	20 ×



23 Oral:

Review of Tables

1. How many in 1, 2, 4, 3, 5, 7, 6 dozen?

2. How many hours in 1, 3, 5, 7, 2, 4, 6 days?

3. How many pints in 1, 4, 6, 5, 2, 3 gallons?

4. How many quarts in 1, 3, 2, 5, 6, 4, 7 gallons?

5. How many ounces in
1
$\frac{1}{2}$ pound?
$\frac{1}{4}$
$\frac{1}{8}$

6. How many inches in
1
$\frac{1}{2}$
$\frac{1}{3}$ yards?
$\frac{1}{4}$
2

7. How many days in
1
3
5
7 weeks?
6
4
2

8. How many cents in
1
5
3
4 dollars?
2
6
$1\frac{1}{2}$

Written:

24 1. How many pint bottles will 15 gallons of milk fill?
2. How many pencils in 12 dozen?
3. What change could the store-keeper give you for a dollar bill?
4. How many school days are there in 18 weeks?
5. How many eggs in 5 dozen?
6. How many times must the farmer use both hands in counting them?
7. How many days in January, February, and March?
8. How many ounces in $2\frac{1}{2}$ lbs.?
9. If I pay 65 cents with a dollar bill, what change shall I receive?
10. Measure the length of the top of your desk in inches. How many feet is that? How many inches over?

LESSON VIII

25 Fractional Parts of Numbers

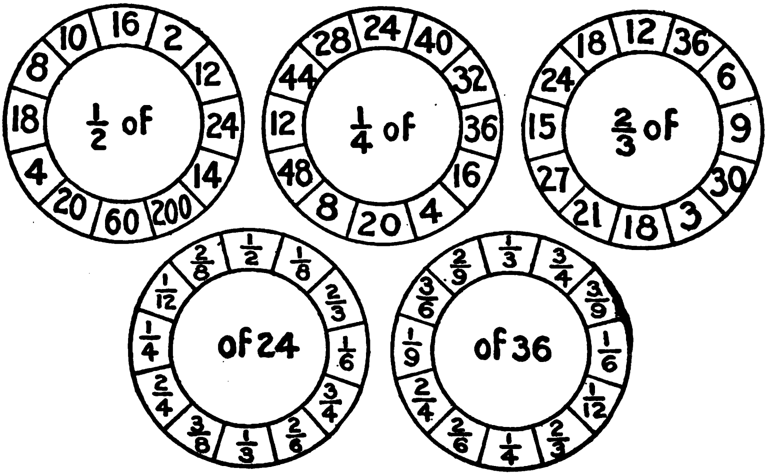
$\frac{1}{3}$ of 12 = 4

$\frac{2}{3}$ of 12 = 2 × 4 or 8

$\frac{2}{\cancel{3}}$ of $\overset{4}{\cancel{12}} = 8$

How many inches long is your ruler? How many inches in $\frac{1}{3}$ of your ruler? In $\frac{2}{3}$ of your ruler? We can say, $\frac{1}{3}$ of 12 = 4. $\frac{2}{3}$ of 12 = 2 × 4 or 8. We can write this in a shorter way. $\frac{1}{3}$ of 12 is 3 divided into 12. Cross out, or *cancel*, the 3 and the 12, and write 4 above 12. $\frac{2}{3}$ is 2 times 4 or 8.

Oral drill:



Written exercise: Cancel:

1.	2.	3.	4.	5.
$\frac{2}{3}$ of	$\frac{3}{4}$ of	$\frac{2}{5}$ of	$\frac{5}{6}$ of	$\frac{3}{8}$ of
6	8	10	18	16
9	16	20	24	8
18	24	15	12	24
24	12	25	30	40
27	20	30	36	32
15	28	45	54	56
	32	50	42	72

Problems

(1) How many inches in $\frac{3}{4}$ of a yard?

(2) How many oranges in $\frac{2}{3}$ of a dozen?

(3) How many hours in $\frac{5}{6}$ of a day?

(4) How many ounces in $\frac{3}{4}$ of a pound?

(5) If $\frac{1}{4}$ of a class of 48 pupils are absent, how many pupils are absent?

(6) A boy who earns $20 a month saves $\frac{2}{5}$ of it. How much does he save?

(7) $\frac{3}{8}$ of a flock of 88 birds were killed. How many were killed?

(8) A man who owned 72 acres of land sold $\frac{2}{9}$ of it. How many acres did he sell?

(9) John has read $\frac{5}{12}$ of a book containing 96 pages. How many pages has he read?

(10) In a crate containing 6 dozen eggs, $\frac{3}{8}$ were cracked. How many were cracked?

(11) Mary had a half-dollar. If she spends $\frac{2}{5}$ of it, how much does she spend?

(12) A man paid $64 for a horse, and sold him for $\frac{5}{8}$ of that price. What did he get for the horse?

(13) $\frac{2}{9}$ of a farm of 81 acres are woodland. How many acres are woodland?

(14) We raised 80 turkeys, and sold $\frac{7}{8}$ of them. How many did we sell?

(15) Our floor contains 40 square yards. If $\frac{5}{8}$ of it is covered, how many square yards are covered?

26 Review—Parts of Numbers

1. 8 is $\frac{1}{2}$
2. 6 is $\frac{1}{3}$
3. 5 is $\frac{1}{4}$
4. 4 is $\frac{1}{8}$

of what number?

5. 7 is $\frac{1}{3}$
6. 6 is $\frac{1}{9}$
7. 5 is $\frac{1}{10}$
8. 3 is $\frac{1}{6}$

of what number?

9. If $\frac{1}{4}$ of a pound of cheese costs 15 cents, what does 1 pound cost?
10. If $\frac{1}{2}$ a dozen eggs costs 16 cents, what does 1 dozen cost?
11. If $\frac{1}{8}$ of a yard of silk costs 25 cents, what does 1 yard cost?
12. If there are 15 pupils in $\frac{1}{3}$ of a class, how many are in the class?
13. If $5 is $\frac{1}{10}$ of the cost of a bicycle, what did the bicycle cost?
14. If 25 pages are $\frac{1}{7}$ of a book, how many pages are in the book?
15. If 35 yards are $\frac{1}{4}$ of the length of the road, how long is the road?

LESSON IX

Short Division

27 1. Operation

$$2\overline{)64}$$
$$32 = \text{quotient}$$

Proof-check

$$\begin{array}{r} 32 \\ \times 2 \\ \hline 64 \end{array}$$

What does this symbol $)\underline{}$ tell us to do? It says to divide 2 into 64, or 6 tens and 4 units. The 2, you know, will divide into the 6 tens three times. Write the 3 under the tens. The 2 will divide into the 4 units twice. Write the 2 under the units. The two will divide into 64, 32 times. The result is called the quotient; the number you divide by is called the divisor; the number you divide into is called the dividend. The way to prove this sort of example is to multiply the quotient by the divisor. The product should be the same as the dividend.

2. Operation

$$3\overline{)7{}^{1}2}$$
$$24 = \text{quotient}$$

Proof-check

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \end{array}$$

What is the dividend? The divisor? What must we do here? The dividend contains how many tens? How many units? The 3 will divide into the 7 tens twice, with one ten over. Write the 2 under the tens. Change the one ten to 10 units, and add these 10 units to the 2 units, making 12 units. Divide the 3 into the 12 units, and write the 4 which you obtain under the units. Prove by multiplying the quotient by the divisor. What should the product be the same as?

3. Operation
3)89
29 = quotient
2 = remainder

Proof-check
29
×3
87
+2
89

What is the dividend? How many tens? how many units? Divide the 3 into the 8 tens. Two times, 2 tens over. Write the 2 under the tens. Change the 2 tens left over to 20 units, and add to the 9 units. Divide the 3 into 29 units. Nine times and 2 units over. Write the 9 under the units. 29 = quotient. 2 = remainder. Prove by multiplying 29 by 3, and adding in the remainder. The result should be the same as the dividend.

Divide:

(1) 63 by 3 (2) 79 by 3 (3) 85 by 5 (4) 468 by 6
(5) 86 by 2 (6) 87 by 4 (7) 90 by 4 (8) 927 by 3
(9) 86 by 4 (10) 96 by 4 (11) 87 by 3 (12) 649 by 4
(13) 75 by 3 (14) 72 by 3 (15) 96 by 2 (16) 275 by 5

4. Operation
3)117
39 = quotient

Here the 3 will not divide into the one hundred. Change it to

10 tens, and add the 10 tens to the 1 ten. Then divide 3 into 11 tens. Three times and 2 over. Write the 3 under the tens. Then divide 3 into 27 units. Nine times. Write the 9 under the units. Prove.

Proof-check

$$\begin{array}{r} 39 \\ \underline{\times 3} \\ 117 \end{array}$$

1. 5)918 2. 7)1248 3. 7)1003 4. 8)2168

5. 9)7439 6. 6)4007 7. 5)91634

8. 8)41368 9. 7)21916 10. 6)41987

Divide:

11. 107 by 3 12. 763 by 7 13. 468 by 5 14. 743 by 9
15. 217 by 5 16. 276 by 6 17. 432 by 7 18. 743 by 8
19. 848 by 4 20. 439 by 8 21. 841 by 8 22. 743 by 7
23. 916 by 7 24. 1276 by 6 25. 649 by 3 26. 743 by 6

27. $\frac{1}{6}$ of	96	=;	28. $\frac{1}{5}$ of	75	=;	29. $\frac{1}{4}$ of	52	=;
	48			90			64	
	102			85			84	
	90			45			92	
	84			65			104	
	120			100			68	
	186			105			44	
	126			95			56	
	243			70			60	
				60			72	

30. $\frac{1}{7}$ of 63 =?
70
112
84
91
98
105
112
119
126
133

31. $\frac{1}{8}$ of 120 =?
114
104
96
128
136
144
152
160
168

32. $\frac{1}{9}$ of 81 =?
90
99
108
117
t12
135
144
153
162
171

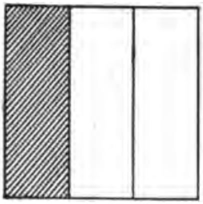
28 Written exercises:

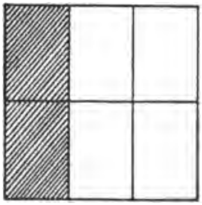
1. How many lbs. of sugar at 6 cts. a lb. can you get for a dollar? Will there be any change?
2. How many dozen eggs are there in 288 eggs?
3. How many weeks are there in 105 days?
4. Find $\frac{1}{9}$ of 11709.
5. Take the quotient in the 4th problem from 4000.
6. The divisor is 8, the dividend is 24864. Find the quotient.
7. The divisor is 9, the dividend is 901717. What is the remainder?
8. What will 25 dozen eggs cost at 35 cents a dozen?
9. How much change will there be after paying for the eggs (in the 8th problem) from ten dollars?
10. How much are 294 bushels, 986 bushels, 241 bushels, 269 bushels?
11. How many days are in 9 years?

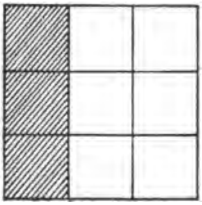
12. In all those days there are how many weeks?
13. Divide 12 into 3996.
14. How many years are there in 4896 months?
15. What is left over when you divide 39089 by 9?
16. 98643 ÷ 8 17. 25 × 9864 18. From 9000 take 6124.
19. 14005 ÷ 7 20. 36 × 2104 21. From 2050 take 987.
22. 29169 ÷ 6 23. 27 × 904 24. From 4002 take 1985.
25. 14004 ÷ 9 26. 63 × 1234 27. From 1000 take XLIII.
28. 9134 ÷ 5 29. 87 × 2193
30. From ten thousand take 4063.

LESSON X

9 One-third and One-sixth to Equivalent Fractions





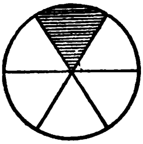


Here are some squares of the same size. The first is divided into how many equal parts? What do we call each part? The shaded part is what part of the whole?

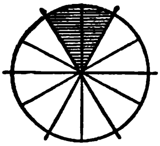
In the second, into how many equal parts is each third divided? The square is divided then into how many equal parts? What do we call each part? The shaded part is how many sixths of the whole? Then $\frac{1}{3} = \frac{2}{6}$.

In the third square, into how many equal parts is each third divided? The square is divided then into how many equal parts? What do we call each part? The shaded part is how many ninths of the whole? Then $\frac{1}{3} = \frac{3}{9}$.

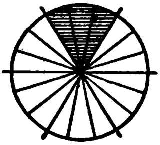
In a similar way, it may be shown that $\frac{1}{3} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18}$.



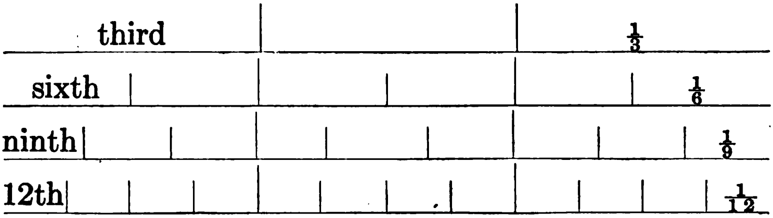
Here are some circles of the same size. Into how many equal parts is the first divided? What do we call each shaded part? The shaded part is what part of the whole?



In the second, into how many equal parts is each sixth divided? The circle, then, is divided into how many equal parts? What do we call each part? The shaded part is how many twelfths of the whole? Then $\frac{1}{6}=\frac{2}{12}$.



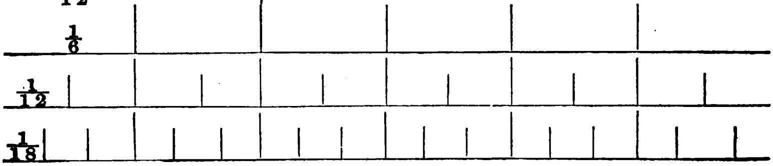
In the third, into how many equal parts is each sixth divided? The circle, then, is divided into how many equal parts? What do we call each part? The shaded part is how many eighteenths of the whole? Then $\frac{1}{6}=\frac{3}{18}$.



1. $\frac{1}{3}$ of the line = how many sixths of the line?
“ “ “ “ = how many ninths of the line?
“ “ “ “ = how many twelfths of the line?

2. $\frac{1}{3}$ of the line = how many times $\frac{1}{6}$ of the line?
“ “ “ “ = how many times $\frac{1}{9}$ of the line?
“ “ “ “ = how many times $\frac{1}{12}$ of the line?

3. $\frac{1}{6}$ of the line
$\frac{1}{9}$ of the line = what part of $\frac{1}{3}$ of the line?
$\frac{1}{12}$ of the line



4. $\frac{1}{6}$ of the line = how many twelfths of the line?
" " " " = how many eighteenths of the line?

5. $\frac{1}{6}$ of the line = how many times $\frac{1}{12}$ of the line?
" " " " = how many times $\frac{1}{18}$ of the line?

6. $\frac{1}{3}=\frac{2}{6}$ $\quad \frac{1}{3}=\frac{3}{9}$ $\quad \frac{1}{3}=\frac{4}{12}$ $\quad \frac{1}{6}=\frac{2}{12}$ $\quad \frac{1}{6}=\frac{3}{18}$

7. $\frac{2}{3}=?$ $\quad \frac{2}{3}=?$ $\quad \frac{2}{3}=?$ $\quad \frac{5}{6}=?$ $\quad \frac{5}{6}=?$

8. How many 12ths in $\left|\begin{array}{l}\frac{1}{2}\\ \frac{1}{3}\\ \frac{2}{3}\\ \frac{1}{4}\\ \frac{3}{4}\\ \frac{1}{6}\\ \frac{5}{6}\end{array}\right.$?

9. How many 16ths in $\left|\begin{array}{l}\frac{1}{2}\\ \frac{1}{4}\\ \frac{3}{4}\\ \frac{1}{8}\\ \frac{3}{8}\\ \frac{5}{8}\end{array}\right.$?

10. How many 18ths in $\left|\begin{array}{l}\frac{1}{2}\\ \frac{1}{3}\\ \frac{2}{3}\\ \frac{1}{6}\\ \frac{3}{6}\\ \frac{5}{6}\end{array}\right.$?

(1) Into how many parts must I cut an orange to get sixths?
(2) How many sixths will there be?
(3) If Mary gets $\frac{3}{6}$ of it, what part of the orange does she get?
(4) If Jane gets $\frac{2}{6}$ of it, what part of the orange does she get?
(5) What part of the orange will be left for John?
(6) Mary's piece is how many times as large as John's?
(7) Jane's piece is how many times as large as John's?
(8) Would you rather have $\frac{1}{3}$ of a pie or $\frac{1}{4}$ of a pie? Why?
(9) Which is more, $\frac{1}{3}$ of a pie or $\frac{2}{6}$ of a pie?
(10) How many cents in one half-dollar?
(11) How many cents in two quarter-dollars?

LESSON XI

THIRD GRADE **SECOND HALF YEAR**

Multiplication

30 Operation

$$\begin{array}{rl} 328 & = \text{multiplicand} \\ \times 257 & = \text{multiplier} \\ \hline 2296 & \\ 1640 & \\ 656 & \\ \hline 84296 & = \text{product} \end{array}$$

$$\begin{array}{ccc} 5\cancel{6} & 4\cancel{0} & 1\cancel{6} \\ 1\cancel{9} & 1\cancel{4} & \cancel{5} \\ 22 & 16 & 6 \end{array}$$

Proof-check

$$\begin{array}{rl} 257 & \\ \times 328 & \\ \hline 2056 & \\ 514 & \\ 771 & \\ \hline 84296 & = \text{product} \end{array}$$

$$\begin{array}{ccc} 5\cancel{6} & 1\cancel{4} & 2\cancel{1} \\ 4\cancel{4} & 11 & 1\cancel{7} \\ 22 & & 5\ 7 \end{array}$$

Read the multiplicand. Read the multiplier. Two hundred fifty-seven means 2 hundreds, 5 tens, 7 units.

Multiply 328 by 7, and place the first figure of the partial product in the units place, under 7. Multiply 328 by 5, and place the first figure of this partial product in the tens place, under 5.

Multiply 328 by 2, and place the first figure of this partial product in the hundreds place, under 2.

Add the partial products. Read the product.

Prove that the product is correct by multiplying the other way:—257 by 328. Always prove your results.

Operation

328
460

19680
1312

150880 = product

Proof-check

460
328

3680
920
1380

150880 = product

Here the multiplier has 0 in the units place. Write 0 in the product in the units place under the 0 in the multiplier. Multiply by 6, and write the first figure in the tens place, under 6. Multiply by 4, and write the first figure in the hundreds place, under 4. Add the partial products. Prove the result.

Operation

328
×507

2296
16400

166296 = product

Proof-check

507
×328

4056
1014
1521

166296 = product

Here the multiplier has 0 in the tens place. Multiply by 7, and place the first figure of the partial product in the units place, under 7. Place 0 in the second partial product in the tens place, under the 0 in the multiplier. Multiply by 5, and place the first figure in the hundreds place, under the 5, and to the left of 0. Add the partial products, and prove as before.

Multiply:

1. 197 by 325
2. 590 by 167
3. 756 by 409
4. 465 by 289
5. 900 by 436
6. 507 by 608

7. 870 by 154 8. 802 by 597 9. 785 by 550
10. 706 by 693 11. 780 by 354 12. 472 by 809
13. 268 by 876 14. 937 by 680 15. 325 by 707

Prove the results.

16. $8.75 ×450
17. $6.50 ×144
18. $4.38 ×375
19. $1.37 ×508
20. $7.65 ×108

31 Problems

(1) How many minutes are in 185 school days of 300 minutes each?
(2) If there are 144 sheets of paper in each bundle, how many sheets are in 250 bundles?
(3) How many feet are in 328 miles of 5280 feet each?
(4) If wheat is bringing $1.15 a bushel, what will 875 bushels bring?
(5) At $2.25 a yard, what will 695 yards of silk cost?
(6) If a barrel of flour weighs 196 pounds, what will 500 barrels weigh?
(7) If a milkman delivers 358 pints of milk each day, how much does he deliver in 180 days?
(8) If a poultry-dealer sells at the rate of 685 pounds of poultry every market day, how many pounds does he sell on 208 market days?
(9) What will 753 tons of coal bring at $7.85 a ton?
(10) What will it cost to supply 980 school children each with a book costing $0.45.

(11) If a man earns $4.50 a day, what will he earn in 278 days?

(12) What will 156 tubs of butter be worth at $7.65 a tub?

(13) If a man takes a trip of 163 miles 125 times, how many miles does he travel?

(14) If it costs $5800 to lay one mile of railroad track, what will it cost to lay 395 miles?

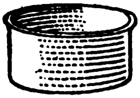
(15) If a hotel uses 350 pounds of ice a day, how many pounds does it use in 365 days?

32

LESSON XII

Dry Measure

Quart



Peck

Bushel

(These measures should be before the class. Sawdust or sand might be used for measuring purposes.)

Which measure is the largest? We use the bushel in measuring grains, like corn; vegetables, like potatoes; and fruits, like apples.

Which measure is the next largest? Fill the peck measure. Empty it into the bushel measure. How many pecks will the bushel measure hold?

What is the name of the smallest measure? This quart measure looks like the quart measure we use for liquids. Fill the quart measure. Find how many quarts make a peck.

Learn this measure, which is called Dry Measure.

8 quarts make 1 peck	8 qt. = 1 pk.
4 pecks make 1 bushel	4 pk. = 1 bu.

Oral:

1. How many quarts will a $\frac{1}{4}$ peck measure hold? A $\frac{1}{2}$ peck measure?
2. How many quarts in 2 bushels? 3 bushels? $\frac{1}{2}$ bushel? $3\frac{1}{2}$ bushels? 5 bushels? 8 bushels?
3. How many pecks in 3 bushels? $\frac{1}{2}$ bushel? $3\frac{1}{2}$ bushels? $\frac{1}{4}$ bushel? $\frac{3}{4}$ bushel? 10 bushels?
4. How many quarts in 3 pecks? 5 pecks? $\frac{1}{2}$ peck? $2\frac{1}{2}$ pecks? $\frac{1}{4}$ peck? $3\frac{1}{4}$ pecks?
5. How many times can the bushel measure be filled from 12 pecks?
6. How many times can the bushel measure be filled from 20 pecks? 32 pecks? 14 pecks (how many pecks over)? 23 pecks?
7. How many times can the peck measure be filled from 5 bushels? 8 bushels? 12 bushels?
8. A quart is what part of a peck? 5 quarts?
9. What is the cost of a bushel of potatoes at $0.30 a peck?
10. If I sell 1 bushel of apples at $0.06 a $\frac{1}{4}$ peck, what do I get for them?

Written:

$$\begin{array}{r} 4 \text{ pk.} = 1 \text{ bu.} \\ \underline{\times 8\phantom{ \text{ pk.} = 1 \text{ bu.}}} \\ 32 \text{ pk.} = 8 \text{ bu.} \end{array}$$

Form to be used:

$$4\overline{)35}\text{ pk.}$$
$$8 \text{ bu.} + 3 \text{ pk.}$$

1. How many pecks of oats in a bin holding 52 bushels?
2. How many pecks of potatoes in a bin holding 38 bushels?
3. If I sell a 2-bushel basket of peaches at $0.32 a peck, how much do I receive?
4. A dealer sold a bushel of onions at $0.15 a quart. How much did he get?
5. How many pecks in 85 bushels? 72 bushels? 97 bushels?
6. What will 2 bushels of chestnuts bring at $0.12 a quart?
7. 15 bushels of oats will last a horse how many days, if he eats a peck a day?
8. Apples are worth $1.92 a bushel. How much is that a peck?
9. John sold 4 pecks of cherries at $0.15 a quart. How much did he get?
10. How many bushels in
    104 pecks?
    320 pecks?
    480 pecks?
    96 pecks?
11. How many pecks in
    136 quarts?
    264 quarts?
    360 quarts?
    312 quarts?

LESSON XIII

Long Division by Numbers of 2 Orders

33 1. Operation

$$\begin{array}{r} 15 = \text{quotient} \\ 13\overline{)195} \\ \underline{13} \\ 65 \\ \underline{65} \end{array}$$

Proof-check

$$\begin{array}{r} 15 \\ \underline{13} \\ 45 \\ \underline{15} \\ 195 \end{array}$$

Here you see we must divide by a number larger than we have studied the multiplication tables for; and since we cannot do so much of the work in our minds we must write out more of it.

13 will not divide into 1 hundred, so divide it into 19 tens. One time and 6 tens over. Write the 1 *over* the tens. Multiply the divisor by 1, and write the product under the 19. Then subtract. The 6 tens which remain equal 60 units; bring down the 5 beside the 6, and we have 65 units. Divide 13 into 65 units. 5 times. Write the 5 *over* the units. Multiply 13 by the 5. Subtract—no remainder. 13 will divide into 195 just 15 times.

Prove by multiplying the quotient by the divisor.

2. Operation

$$\begin{array}{rl}
\overline{32} & =\text{quotient}\\
14)\overline{448} & \\
\underline{42} & \\
28 & \\
\underline{28} &
\end{array}$$

Proof-check

$$\begin{array}{r}
32\\
\underline{\times 14}\\
128\\
\underline{32}\\
448
\end{array}$$

14 will divide into 44 three times. Write the 3 over the 4. $3 \times 14 = 42$. Subtracting will give remainder 2. Bring down the 8 beside the 2. $28 \div 14 = 2$. Write the 2 over the 8. Multiply and subtract as in the first case.

How will you prove your work in division?

3. Operation

$$\begin{array}{rl}
\overline{44} & =\text{quotient}\\
15)\overline{669} & \\
\underline{60} & \\
69 & \\
\underline{60} & \\
9 & =\text{remainder}
\end{array}$$

Proof-check

$$\begin{array}{r}
44\\
\underline{15}\\
220\\
\underline{44}\\
660\\
\underline{+\quad 9}\\
669
\end{array}$$

If there should be a remainder in long division, you must add it to the product of the quotient and divisor when you prove your work.

What should this product + the remainder be the same as?

4. Operation

$$\begin{array}{r} 240 = \text{quotient} \\ 25\overline{)6000} \\ \underline{50} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

Proof-check

$$\begin{array}{r} 240 \\ \underline{\times 25} \\ 1200 \\ \underline{480} \\ 6000 \end{array}$$

We must be careful not to make the number in the quotient too large, and we must never have a remainder as large as the divisor. If we make either of these mistakes we must try again.

Here 25 will divide into 60 two times; into 100 four times. This time there is no remainder. Bring down the 0. 25 will not divide into the 0. So write a 0 in the quotient over the 0 in the dividend.

Divide:

1. 250 by 16
2. 1768 by 24
3. 2525 by 25
4. 287 by 17
5. 6014 by 19
6. 4848 by 24
7. 408 by 15
8. 2168 by 22
9. 9608 by 16
10. 516 by 18
11. 4190 by 25
12. 4928 by 42
13. 21005 by 75
14. 3926 by 13
15. 64587 by 31
16. 4986 by 56
17. 5684 by 28
18. 41268 by 27
19. 4129 by 29
20. 7650 by 75
21. 41003 by 62
22. 50450 by 25
23. 14093 by 19
24. 149814 by 37

Find:

25. $\frac{2}{5}$ of 65
26. $\frac{1}{9}$ of 81666
27. 29×6194
28. $\frac{4}{7}$ of 84
29. $\frac{1}{8}$ of 72864
30. 28×4169
31. $\frac{5}{9}$ of 81
32. $\frac{1}{7}$ of 71715
33. 78×4196
34. $\frac{6}{7}$ of 427
35. $\frac{1}{6}$ of 61956
36. 95×2064
37. $\frac{8}{9}$ of 909
38. $\frac{1}{8}$ of 816914
39. 56×9073
40. 69×7895

Write answers in words.

Problems

(1) If $550 is divided among 25 people, how much will each get?

(2) If a yard of silk costs $0.95, how many yards can you get with $19.00?

(3) How many quarter dollars are worth $7.75?

(4) If a pound of butter costs 36 cents, how many pounds can you get with 75 cents? Will there be any change?

(5) If a railroad train runs 32 miles an hour, how long will it take it to run 196 miles?

(6) What is the perimeter of a 36-foot square?
How many yards in this perimeter?

(7) How many yards in 10836 inches?

(8) How many years in 576 months?
How many days in these years?

(9) An oblong is 9 yards wide, 15 yards long. What is its perimeter?

(10) How many inches in this perimeter?
(11) Divide seven hundred, sixty-eight by XXIV.
(12) Divide 1956 by sixteen.
(13) Take 9243 from ten thousand, forty.
(14) Add 296 inches, 403 inches, 1930 inches, 493 inches, 1409 inches.
(15) How many yards will the last answer be?

(16) $96413 \div 75$ (17) $9124 - 987$
(18) $21064 \div 84$ (19) $6001 - 4291$
(20) $17614 \div 19$ (21) $17603 - 8926$
(22) 49×619 (23) $\frac{4}{5}$ of 955
(24) 56×287 (25) $\frac{2}{3}$ of 1653
(26) 53×1986 (27) $\frac{4}{7}$ of 1561

LESSON XIV

Review—Drill—Counting

34 Oral:

1. How many sq. in. in
1
$\frac{1}{2}$
$\frac{1}{3}$
$\frac{1}{4}$
$\frac{1}{6}$
$\frac{1}{8}$
$\frac{1}{9}$
$\frac{1}{12}$
2
sq. ft.?

2.
$\frac{2}{3}$
$\frac{5}{8}$
$\frac{4}{6}$
$\frac{11}{12}$
$\frac{1}{2}$
$\frac{5}{6}$
$\frac{7}{8}$
$\frac{3}{4}$
of 24 = ?

3. How many
sixths
eighths
tenths
fourths
twelfths
fourteenths
sixteenths
eighteenths
twentieths
= $\frac{1}{2}$?

4. 1 bushel = how many
pecks?
quarts?
pints?

5. How many months in
1
2
3
4
5
$\frac{1}{2}$
$\frac{1}{4}$
$\frac{3}{4}$
$\frac{5}{6}$
$\frac{2}{3}$
years?

6.
7 weeks
9 "
10 "
6 "
12 "
14 "
15 "
8 "
6 "
3 "
= ? days

7. 2 bushels	= how many	pecks?
8. 3 "		quarts?
9. ½ bushel		pints?
10. ¼ "		
11. 5 bushels		

12.	9		1	
	1		12	
	12		2	
	7		11	
	6		3	
	8		9	= how
	11	×	4	many?
	4		8	
	3		5	
	2		7	
	5		6	
	10		10	

Get rapid answers here.

What is the remainder when you divide:

13. 65 by 8, 9, 2, 3, 4, 5, 6, 7
14. 97 by 9, 8, 2, 3, 4, 5, 6, 7
15. 69 by 4, 5, 6, 7, 8, 2, 3, 9
16. 72 by 5, 6, 7, 8, 9, 2, 3, 4
17. 84 by 6, 7, 8, 9, 2, 3, 4, 5
18. 29 by 2, 3, 4, 5, 6, 7, 8, 9
19. 47 by 3, 4, 5, 6, 7, 8, 9, 2
20. 56 by 7, 8, 9, 2, 3, 4, 5, 6

9
6
5
4
8
7
2
1
4
7
8
9
4
3
6
2

Beginning at the 9, then at the 6, and so on, add rapidly down the column.

Beginning at the 2, then at the 6, add up the column. Change the order of the figures in the column frequently.

Require *rapidity*.

NOTE.—*No. 12* is another of the drills in multiplication. It introduces every combination up to 12 × 12, using each figure in the first column as a multiplier of every figure in the second column.

21. How many:
gills | pints | quarts in
1 gal.
2 gal.
3 gal.
4 gal.
2 qts.
7 qts.
16 pts.
8 pts.
12 pts.
2 pts.
$\frac{1}{2}$ gal.
$\frac{1}{4}$ gal.
$\frac{2}{4}$ gal.
$\frac{3}{4}$ gal.

22. $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{1}{8}$ | $\frac{2}{8}$ | $\frac{3}{8}$ | $\frac{4}{8}$ | $\frac{5}{8}$ | $\frac{6}{8}$ | $\frac{7}{8}$ of 16 qts. = how many pints, quarts, or gallons?

23.
1 lb.
$\frac{1}{2}$ "
$\frac{1}{4}$ "
$\frac{2}{4}$ "
$\frac{3}{4}$ "
$\frac{1}{8}$ "
$\frac{2}{8}$ "
$\frac{3}{8}$ "
$\frac{5}{8}$ "
$\frac{7}{8}$ "
= how many ounces?

24.
8 oz.
4 "
12 "
2 "
1 "
5 "
7 "
9 "
10 "
3 "
15 "
= what part of a pound?

35 Written:

1.	2.	3.	4.	5.	6.
9143	1234	7163	2198	973	2984
+6197	+7197	+1459	+1793	+1986	+199
1278	7913	1769	9876	67	1976
7642	6145	1237	5432	413	1208
9137	6589	1986	9164	419	794
2163	1745	2197	1276	9864	618

Write in words each addend and each sum.

7.	**8.**	**9.**	**10.**	**11.**	**12.**
100016	91684	219640	21098	21498	60103
−98724	−1979	−19994	−1799	−1719	−40198

Write in words each minuend, subtrahend, remainder.

13.	**14.**	**15.**	**16.**	**17.**	**18.**
9864	41989	61496	42987	9999	90976
×9	×7	×8	×5	×9	×8

Write in words each multiplicand and each product.

19. Multiply 6945 by 97; 64; 28; 57; 90; 53; 62. Add the products.

Divide:

20. 98654 by 9, 8, 7, 6, 5
21. 21006 by 7, 8, 9, 5, 6
22. 21707 by 9, 8, 7, 6, 5 | Prove each.
23. 10919 by 9, 8, 7, 6, 5
24. 29463 by 9, 10, 11, 12, 8

25. 191643 ÷ 16
26. 210764 ÷ 75
27. 210764 ÷ 64 | Prove each.
28. 910632 ÷ 48
29. 214307 ÷ 27
30. 416891 ÷ 74

31. Multiply:

Seven thousand, four hundred, sixteen
Nine hundred, fifty
One thousand, sixty-four
Fifty-nine thousand, three hundred, nine

by

XXIV
XLV
XXXVIII
XIX
XXV
XVI
XXVII
XLVIII
L
IX

Write each product in words.

Oral:

Count by 5's to 100, beginning at 0, 1, 2, 3, etc.
" " 10's to 100.
" " 4's, 5's, 6's, 7's, 8's, 9's to 100, beginning at 0, 1, 2, 3, etc.

Count back from 100 by 3's, 4's, 5's, 6's, 7's, 8's, 9's, 10's.

4×15		9×300		12× 12	
5×15		9×700		12×120	
6×13		6×500		11× 11	
7×13		7×500		11×110	
8×13	=?	8×400	=?	10× 9	=?
6×15		8×600		10× 90	
7×20		5×900		9× 8	
8×20		5×800		9× 80	
9×20		5×700		9×800	

How many units in

17 tens
65 tens
98 tens

How many tens in

6 hundreds
16 hundreds
63 hundreds
4 hundreds
45 hundreds
97 hundreds
1 thousand
10 thousands
12 thousands

LESSON XV

Long Measure

36 We have measured the length of our rulers and books by inches, the length of the desk by feet, and the length of the floor by yards. We need a still longer measure for the length of streets, roads, railway tracks, and distances between cities. This long measure we call a mile. Let us see how long it is.

12 inches = 1 foot
3 feet = 1 yard
1760 yards = 1 mile

1 mile = 1760 yards
" " = 5280 feet

Oral drill:

1. 3
2
5
4
6
8
7
9
feet = how many inches?

2. 10
12
11
9
20
8
6
30
42
yards = how many feet?

3. $\frac{1}{2}$
$\frac{1}{3}$
$\frac{1}{4}$
$\frac{1}{6}$
$\frac{1}{12}$
$\frac{2}{3}$
$\frac{3}{4}$
of a foot = how many inches?

4. $\frac{1}{2}$
$\frac{1}{3}$
$\frac{1}{4}$
$\frac{1}{6}$
$\frac{1}{9}$
$\frac{1}{12}$
$\frac{4}{9}$
of a yard = how many inches?

5. 24
48
60
75
39
86
100
53
inches = how many feet and how many inches?

6. 27
36
39
18
20
38
49
52
65
feet = how many yards and how many feet?

Written exercise. Form to be used:

$$\begin{array}{rl} 1760 \text{ yd.} & = 1 \text{ mile} \\ \times 5 & \\ \hline 8800 \text{ yd.} & = 5 \text{ miles} \end{array} \qquad \begin{array}{rl} 5280 \text{ ft.} & = 1 \text{ mile} \\ \times 8 & \\ \hline 42240 \text{ ft.} & = 8 \text{ miles} \end{array}$$

7. 11
30
26
5
13
24
miles = ? yards

8. 4
6
8
10
25
36
miles = ? feet

9. $\frac{1}{2}$
$\frac{1}{3}$
$\frac{1}{4}$
$\frac{1}{6}$
$\frac{1}{8}$
$\frac{1}{12}$
mile = ? feet

10. $\frac{2}{3}$
$\frac{3}{4}$
$\frac{3}{6}$
$\frac{5}{8}$
$\frac{7}{10}$
mile = ? feet

11. If 36 inches make 1 yard, how many yards in 324 inches? 972 inches? 1188 inches? 900 inches? 2088 inches?

12. How many yards and how many inches over in 1098 inches? 1825 inches? 545 inches? 753 inches? 1760 inches?

Problems—Oral

(1) Our bookcase is 8 feet high. How many yards and feet high is it?

(2) A hall is 8 yards long. How many feet long is it?

(3) A boy had a string 12 feet long. If he cut off $\frac{1}{3}$ of it, how many inches did he cut off?

(4) A flower bed is 2 yards long, 1 yard wide. The distance around it is how many feet?

(5) If it is 12 yards from your door to the store, how many feet do you walk when you go there and return?

Problems—Written

(1) Some men had a mile of road to repair. How many feet had they finished when they had done $\frac{3}{4}$ of it?

(2) How many feet of track in a double-track railway 17 miles long?

(3) A man started out to walk a mile, but stopped after going 985 yards. How many yards were left in the mile?

(4) If a mountain is 5 miles high, how many feet high is it?

(5) If the city builds 48 yards of road a day, how many days will it take to build a mile?

(6) If a river is 291 feet wide, how many yards long must a bridge across it be?

(7) How many inches around a table-cover 5 feet long, 4 feet wide?

(8) How many feet around a garden 36 feet long and $\frac{2}{3}$ as wide?

(9) If it is a half mile from your house to the station, how many minutes will it take you to walk there at the rate of 132 feet each minute?

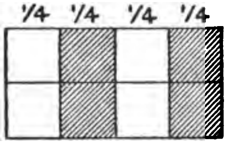
(10) How many feet of fencing will enclose a lot 25 yards long, 18 yards wide?

LESSON XVI

Equivalent Fractions

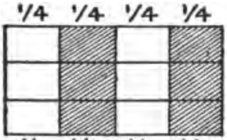
37 (Show an oblong, divided by vertical lines into 4 equal parts.)

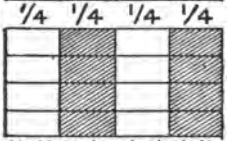
This oblong is divided by vertical lines into 4 equal parts. What do we call each part? If we divide each fourth into two equal parts (by the horizontal), how many equal parts will there be? What part of the whole oblong will each part be? How many of these eighths will there be in each fourth? $\frac{1}{4}$ of a thing $= \frac{2}{8}$ of the thing.



(And so on, by diagrams and progressive division have the pupils see that

$$\begin{aligned}\tfrac{1}{4} \text{ of a thing} &= \tfrac{3}{12} \text{ of the thing}\\ &= \tfrac{4}{16} \text{ of the thing}\\ &= \tfrac{5}{20} \text{ of the thing)}\end{aligned}$$

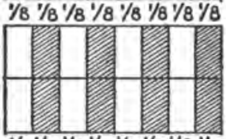


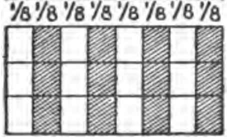


(Show an oblong divided into 8 equal parts by vertical lines.)

What is each part called? Divide each eighth into two equal parts (by horizontal line). How many of these equal parts in the oblong? What part of the whole oblong is each part? How many of these sixteenths in each eighth?

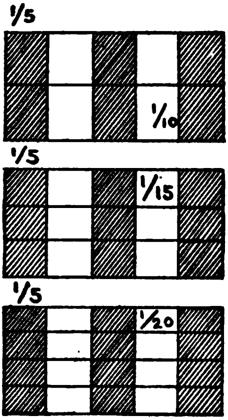
$\frac{1}{8}$ of a thing $= \frac{2}{16}$ of the thing.





By diagram and progressive division have the pupils see that $\frac{1}{8}$ of a thing $=\frac{3}{24}$ of the thing.

Also in similar ways show that



$\frac{1}{5}$ of a thing $=\frac{2}{10}$ of the thing
$=\frac{3}{15}$ of the thing
$=\frac{4}{20}$ of the thing

Oral:

$\frac{1}{4}$ = how many eighths
twelfths
sixteenths
twentieths

$\frac{1}{8}$ = how many sixteenths
twenty-fourths

$\frac{1}{5}$ = how many tenths
fifteenths
twentieths

1. $\frac{1}{4}$ of 100 cents
2. $\frac{2}{8}$ of 48 pounds
3. $\frac{3}{12}$ of 36 eggs
4. $\frac{4}{16}$ of 80 trees
5. $\frac{5}{20}$ of 88 cents
6. $\frac{1}{8}$ of 72 dozen | = how many?
7. $\frac{2}{16}$ of 64 days
8. $\frac{3}{24}$ of 96 years
9. $\frac{1}{5}$ of \$20
10. $\frac{2}{10}$ of \$100
11. $\frac{3}{15}$ of \$75
12. $\frac{4}{20}$ of 50 marbles

13. How many
eighths
twelfths
sixteenths
twentieths
in $\frac{1}{4}$?

14. How many
sixteenths
twenty-fourths
in $\frac{1}{8}$?

15. How many tenths
fifteenths
twentieths
in $\frac{1}{5}$?

16. $\frac{1}{4} = \frac{?}{8}$
$\frac{1}{5} = \frac{?}{10}$
$\frac{1}{4} = \frac{?}{12}$
$\frac{1}{4} = \frac{?}{20}$

17. $\frac{1}{4} = \frac{?}{16}$
$\frac{1}{5} = \frac{?}{15}$
$\frac{1}{5} = \frac{?}{20}$
$\frac{1}{8} = \frac{?}{16}$
$\frac{1}{8} = \frac{?}{24}$

18. $\frac{1}{4}$ of 16
$\frac{2}{4}$ of 16
$\frac{1}{8}$ of 16
$\frac{2}{8}$ of 16
$\frac{3}{8}$ of 16
$\frac{5}{8}$ of 16
= ?

Written:

(1) John had 100 marbles; Tom had $\frac{5}{20}$ as many. How many had Tom?

(2) A man sold 510 lbs. of sugar one week; the next week he sold only $\frac{3}{15}$ as much. How many pounds did he sell the second week?

(3) Which is more, $\frac{3}{12}$ of 60 or $\frac{3}{15}$ of 45? How much?

(4) Take $\frac{3}{24}$ of 96 from one hundred one.

(5) Show that $\frac{1}{5} = \frac{4}{20}$.

(1) Take 69 from $\frac{4}{16}$ of 400.

(2) Add $\frac{1}{4}$ of 32, $\frac{4}{16}$ of 48, $\frac{5}{20}$ of 100, $\frac{2}{16}$ of 40, $\frac{3}{24}$ of 80.

(3) If 1 yard of cloth costs $1.68, what will $\frac{3}{24}$ of a yard cost?

(4) If a man travels 168 miles in a day, how far will another travel who goes $\frac{5}{6}$ as far?

(5) $\frac{1}{4}$ of 96 = $\frac{1}{5}$ of 120 = $\frac{3}{24}$ of 56 = Add these
$\frac{2}{8}$ of 108 = $\frac{4}{20}$ of 65 = $\frac{1}{8}$ of 96 = answers.

(1) If corn meal costs $\frac{8}{16}$ as much as wheat flour, what will 10 pounds of meal cost if 10 pounds of flour cost 48 cents?

(2) How much more is $\frac{5}{15}$ of 96 than $\frac{8}{16}$ of 56?

(3) To seven hundred eleven add $\frac{3}{12}$ of 712.

(4) Divide 1225 by $\frac{5}{15}$ of 45.

(5) $694+386+927+\frac{4}{16}$ of $888=$?

LESSON XVII

Roman Numerals

38 There are two ways of writing numbers. The one called the Arabic system uses ten figures,—1, 2, 3, 4, 5, 6, 7, 8, 9, 0. The one called the Roman system uses seven letters, having the following values:

I = 1 X = 10 C = 100 M = 1000
V = 5 L = 50 D = 500

We have already seen the Roman numerals from I to XII on the clock face. Now we shall see how these numbers were made, and learn to make others.

When letters of equal value are placed side by side, add the value of the letters to get the value of the number; thus, III = 3.

When letters of unequal value are placed side by side, if the smaller value comes first, subtract the value of the smaller from the value of the larger; thus IV = 4 (1 less than 5).

When letters of unequal value are placed side by side, if the larger value comes first, add the value of the smaller to the value of the larger; thus VI = 6 (1 more than 5).

1 = I		11 = XI	10 + 1
2 = II		12 = XII	10 + 2
3 = III		13 = XIII	10 + 3
4 = IV	1 from 5	14 = XIV	10 + 4
5 = V		15 = XV	10 + 5
6 = VI		16 = XVI	10 + 6
7 = VII		17 = XVII	10 + 7
8 = VIII		18 = XVIII	10 + 8
9 = IX	1 from 10	19 = XIX	10 + 9
10 = X		20 = XX	10 + 10

Write the numbers from 20 to 29

30 = XXX		30 to 39
40 = XL	(10 less than 50)	40 to 49
50 = L		50 to 59
60 = LX	(50 + 10)	60 to 69
70 = LXX		70 to 79
80 = LXXX		80 to 89
90 = XC	(10 less than 100)	90 to 99

100 = C			
200 = CC	(100 + 100)	600 = DC	(500 + 100)
300 = CCC		700 = DCC	
400 = CD	(100 less than 500)	800 = DCCC	
500 = D		900 = DCCC	
		1000 = M	

Read these numbers:

1. XIX XCV LXXIII LIII XVIII
2. XXXIV LXXXI XII LXVII XXIII
3. XLV CXIX XXVIII XCVI LVIII
4. CXL CCL XXXVI CIII LXXIX
5. LXXII XLIV XLIX DCX XV

Write in Roman numerals:

1. 27 59 18 76 109
2. 45 63 33 81 125
3. The number of days in January, February, April.
4. The day of the month to-day is.
5. The number of days in a week.
6. The number of school days in a week.
7. The number of weeks in a year.
8. The number of months in a year.
9. The number of years in your age.
10. The number of pages in your book.

Problems

(1) How much more is C than LVIII?

(2) Add XXIX, XLIV, LXVI, XIX, XXXIII. Write the answer in Roman numerals.

(3) How many more pages has a book of CCCXII pages than one of CCXXVIII pages?

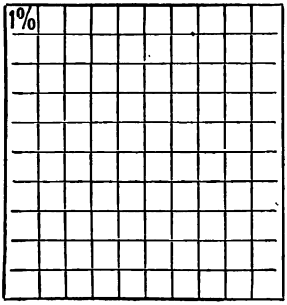
(4) Multiply CCLIX by CXCV.

(5) How many chapters between Chapters XXXVIII and LV?

LESSON XVIII

39

Percentage



(Show a square divided into 100 equal parts.) This square is divided into 10 equal rows. What part of the whole square is each row? Each row is divided into 10 equal parts. How many of the small parts are in the square? Each part is what part of the whole square?

How many hundredths in 1 row, or $\frac{1}{10}$ of the square?

$$\frac{10}{100} = \frac{1}{10}$$

How many hundredths in 5 rows, or $\frac{1}{2}$ of the square?

$$\frac{50}{100} = \frac{1}{2}$$

How many hundredths in $2\frac{1}{2}$ rows, or $\frac{1}{4}$ of the square?

$$\frac{25}{100} = \frac{1}{4}$$

The word *percentage* means *by the hundred,* and we use it when we reckon by the hundred—when we say $\frac{1}{100}$ of a thing, $\frac{10}{100}$ or $\frac{1}{10}$ of a thing, $\frac{50}{100}$ or $\frac{1}{2}$ of a thing. We call $\frac{1}{100}$, one per cent, and write it this way, 1%. (Note the new symbol.)

How many per cent of the square in 1 row? 2 rows? 3 rows? 4 rows? 5 rows? 10 rows?

How many per cent in 1 row and 5 small squares? 2 rows and 7 small squares?

How many % in | 6 rows and 3 small squares?
7 rows and 5 small squares?
10 rows, or all of the big squares?

One little square = what % of the whole?
Two little squares = what % ?
Ten little squares = what % ?
Fifty little squares? One row? Five rows?
Seven rows? Seven rows and five squares?
Nine rows? Ten rows?
All of a thing = what % of the thing?

$10\% = \frac{10}{100}$ $\frac{10}{100} = \frac{1}{10}$ $10\% = \frac{1}{10}$

(Let the pupils reach these results in answers to questions.)

What is 10% of \$50? of \$60? of \$90?
What is 10% of 40 bushels? of 60 bushels? of 100 bushels?
What is 10% of 100 days? of 120 days? of 150 days?
What is 10% of 360 inches? of 70 inches? of 450 inches?
What is 10% of 80 boys? of 500 boys? of 400 boys?

$10\% = \frac{1}{10}$

$50\% = \frac{50}{100}$ $\frac{50}{100} = \frac{1}{2}$ $50\% = \frac{1}{2}$

(Let the pupils reach these results in answers to questions.)

$50\% = \frac{1}{2}$

What is 50% of $500? of $48? of $96? of $150?

What is 50% of 80 miles? 54 miles? 36 miles? 50 miles?

What is 50% of 120 trees? 60 trees? 30 trees? 72 trees?

What is 50% of 74 pounds? 88 pounds? 98 pounds? 68 pounds?

What is 50% of 48 years? 190 years? 500 years? 600 years?

$25\% = \frac{25}{100}$ $\frac{25}{100} = \frac{1}{4}$ $25\% = \frac{1}{4}$

(Let the pupils reach these results in answers to questions.)

$25\% = \frac{1}{4}$

What is 25% of $40? $80? $120? $96?

What is 25% of 400 miles? 600 miles? 300 miles? 800 miles?

What is 25% of 24 inches? 48 inches? 72 inches? 100 inches?

What is 25% of 16 pints? 160 pints? 1600 pints? 16000 pints?

What is 25% of 28 boys? 44 boys? 56 boys? 72 boys?

Written exercises:

Form to be followed:

$25\% = \frac{25}{100}$

$\frac{25}{100} = \frac{1}{4}$

$\frac{1}{4}$ of $\cancel{\$952} = \$238$

$238 = answer

1. What is 25% of $952?
2. A man had 68 hens and lost 50% of them. How many were lost?
3. If there are 180 school days in a year, and Tom stays home 10% of the time, how many days does he lose?
4. There were 160 pages in the book. We had studied 25% of them. How many had we studied?
5. Which is greater, 50% of $200 or 25% of $164?

Find 50% of	4264	Find 25% of	6464	Find 10% of	750
	5000		7284		6940
	7496		9152		8060
	988		892		4930
	406		1696		780
	1960		196		1620

NOTE.—Oral problems in buying and selling, involving the simple per cents, will always prove interesting, are endless in variety and number, and are the best means of making clear the rudiments of percentage.

Seat-work suggestions: Make a line 50% longer than a foot. How long is it?

Make a square 4 in. × 4 in. How many square inches does it contain?

Make one 25% longer and wider. How many square inches does it contain?

Make one 50% longer and wider. How many square inches does it contain?

Make one 100% longer and wider. How many square inches does it contain?

Make diagrams illustrating per cent.

Shade parts of the diagram to illustrate 10%, 20%, 30%, 25%, 65%, etc., 67%, 91%, etc.

LESSON XIX

Long Division by Numbers of Three Orders

40 Operation

$$\begin{array}{rl}
\underline{5578} & = \text{quotient} \\
112)\overline{624736} & \\
\underline{560} & \\
647 & \\
\underline{560} & \\
873 & \\
\underline{784} & \\
896 & \\
896 &
\end{array}$$

Proof-check

$$\begin{array}{r}
5578 \\
\underline{\times 112} \\
11156 \\
5578 \\
\underline{5578} \\
624736
\end{array}$$

Read this dividend. Read the divisor. What will be the name of the answer? (Quotient.) The numbers are long, so we must find the quotient part by part. 112 will not divide into 6, or 62; so there will not be a figure in the quotient over 6 or 2. How many times will it divide into 624? 5 times. Write the 5 over the 4; multiply the divisor by 5; then subtract; bring down the 7. Divide 112 into 647. 5 times. Write the 5 over the 7; multiply the divisor by 5; subtract, bring down the 3. Divide 112 into 873. 7 times. Write the 7 over the 3; multiply the divisor by 7; subtract; bring down the 6. Divide 112 into 896. 8 times. Write the 8 over the 6; multiply the divisor by 8; then subtract—no remainder—even division. Read the whole quotient.

Prove by multiplying quotient by divisor. What should the product be the same as? Always prove—so that you will *know* yourself that there are no errors in the operation.

Operation

```
        68504 =quotient
528)36170117
    3168
    ----
     4490
     4224
     ----
      2661
      2640
      ----
        2117
        2112
        ----
           5 =remain-
                 der
```

Proof-check

```
   68504 =quotient
   ×528 =divisor
  ------
  548032
 137008
342520
--------
36170112
       5 =remainder
--------
36170117
```

Read this big dividend. Read the divisor. The quotient will be long; so we must find it part by part. 528 will not divide into 3, or 36, or 361. How many times will it divide into 3617? 6 times. Write the 6 over the 7; multiply the divisor by it; then subtract; bring down the 0. Divide again. 8 times. Write 8 over the 0; multiply the divisor by it; subtract; bring down the 1. Divide again. 5 times. Write the 5 over the 1. Multiply; subtract; bring down the 1. You can not divide 528 into 211. Write a 0 in the quotient over the 1; bring down the 4. Divide 528 into 2117. 4 times. Write the 4 over the 7; multiply; subtract; the remainder is 5. Read the quotient. Prove by multiplying quotient by divisor, and adding the remainder to the product. What should the result be the same as?

Divide:

1. 62545 by 145
2. 61413 by 213
3. 49864 by 173
4. 50050 by 150
5. 2198614 by 182
6. 1701456 by 196
7. 2178917 by 168
8. 1986400 by 250
9. 1801040 by 123
10. 468914 by 178
11. 1432107 by 109
12. 2168719 by 684

1. How many years are there in 12345 days?
2. How many men could receive $375 from $10000?
3. If a steamer travels 315 miles a day, how long will it take it to cross 3000 miles of sea?
4. It is 98 miles from Baltimore to Philadelphia. How many times must an engine run that distance to run 7050 miles?
5. A half-mile is 880 yards. How many half-miles in 15480 yards?

To the pupil:

A good way to prove multiplication is to divide the product by either multiplicand or multiplier. The result should be the same as the multiplier or multiplicand.

Find:
1. 689 × 4698
2. 608 × 5697
3. 649 × 10683
4. 453 × 9175
5. 812 × 76193
6. 245 × 876
7. 407 × 12345
8. 619 × 21014
9. 613 × 98761
10. 419 × 20140
11. 271 × 8976
12. 413 × 817
13. 684 × 917
14. 615 × 916
15. 436 × 274
16. 416 × 125
17. 375 × 264
18. 123 × 987

Prove each example by dividing product by multiplier.

1. How many square feet are there in 5184 square inches?
2. How many square inches in 150 square feet? Prove this by division.
3. How many yards in 999972 inches?
4. Add 259, 259, 518, 777, 1036, 2590. Write the sum in words.
5. Divide the answer in the 4th by 259. Prove the answer.

LESSON XX

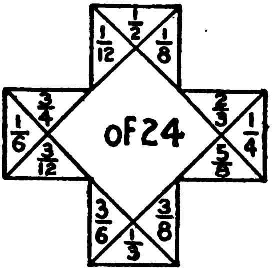
General Review

41 Oral:

Beginning at	1 3 5	7 9 8	6 4 2	count by	9's 6's 3's	8's 4's 7's	5's 2's	to 100

Beginning at 100, count backward by	10's 5's 8's	6's 4's 2's	7's 9's 3's

1. How many	2's 4's 6's 3's 9's 12's	in 36?
2. How many	10's 4's 8's 5's 2's	in 40?
3. How many	8's 9's 6's 12's 4's 3's	in 72?



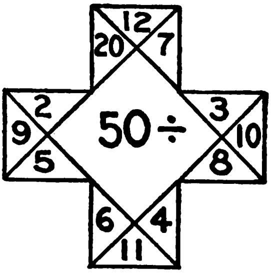
$\frac{1}{2}$ = how many 4ths? 8ths? 12ths? per cent?

$\frac{1}{4}$ = how many 8ths? 12ths? per cent?

$\frac{1}{5}$ = how many 10ths? 20ths? per cent?

$\frac{1}{10}$ = how many per cent?

4. $5+8+9+7+6=?$
5. $12+6+8+9+7=?$
6. $8+9+7+5+6-10=?$
7. $10+8-9+7-8+8=?$
8. $20+8-5-9=?$
9. $15-7+12-6=?$



Read these numbers:

1,897
908
30,050
7,809
10,537
325,080
1,050,950

XVI
XXXIX
XLVII
XIV
LXXXV

1. How many more than half a dozen are 10? How many less than a dozen?
2. How many days are there in April and May?
3. If a quart of milk costs 9 cents, what will one gallon cost?
4. If $\frac{2}{5}$ of a class of 50 pupils are absent, how many are present?
5. Where are the hands of a clock at 3 o'clock? 6 o'clock? 12 o'clock?
6. How much was received for a peck of chestnuts sold at $0.12 a quart?
7. How many bananas on a stalk containing $7\frac{1}{2}$ dozens?
8. How many hours do you spend in school a day? How many minutes?
9. If I pay for 3 yards of 25-cent material with a $1 bill, what change do I get?
10. 56 days are how many weeks?

11. If a man received \$4.80 for 8 hours' work, how much did he earn per hour?
12. How many square inches could you cut from a paper 12 inches long, 11 inches wide?
13. If a man who earns \$120 a month saves 25% of it, how much does he save?
14. What is left from a quarter-dollar after paying for ½ pound of coffee at 36 cents a pound?
15. What part of a week are 2 days? What part of a day are 12 hours?
16. What do 4 bicycles cost when 1 costs \$55?
17. How many pounds of sugar at 6 cents can be bought for \$0.54?
18. A rug 5 yards long, 3 yards wide covers how many square feet of floor?
19. John has 20 marbles. James has 12. How many has each after John gives James 5?
20. How many years have passed since 1904?
21. How much money in 2 quarters, 2 dimes, 2 nickels, 2 cents?
22. John ate ½ of a pie, James ate ¼ of it. How many fourths of the pie were left?
23. In a crate of 8 dozen eggs, only 3 were cracked. How many were whole?
24. In how many ways can you make the number 24?
25. If 8 miles are ¼ of the distance to town, what is the distance?

Written:

	Grades							
	1	2	3	4	5	6	7	8
Monday	42	48	50	45	40	46	33	29
Tuesday	45	46	49	43	43	43	31	30
Wednesday	40	47	48	45	39	45	29	28
Thursday	39	43	49	44	42	44	32	29
Friday	43	45	47	40	38	46	30	27

1. Add across and get the total attendance for each day.
2. Add down, and get the total attendance for each grade.
3. Add the totals of the grades (across).
4. Add the totals for the days (down). Are the sums equal?
5. How much greater was the attendance on Monday than on every other day?
6. How much greater was the 3rd grade attendance for the week than that of every other grade?

Multiply	87	by	95
	965		480
	309		507
	150		392
	876		706
	246		890

Prove results by division.

Divide

2769	72008	3756	19753
8072	12060	55673	
66213	6935		

each by

695	49	987	92	735	38

Prove results.

Add:

1. $2.15	2. $3.50	3. $1.35
.87	.92	.62
.35	.85	2.37
3.75	4.16	3.80
.63	3.29	.38
9.05	.75	5.75

Prove results.
How much greater is each sum than $10?
How much less than $20?

1. How many acres in a farm containing 279 acres of woodland, 159 acres of pasture land, and 36 acres of orchard?
2. A horse was sold for $3800, which was $675 more than was paid for it. How much was paid for it?
3. How many bushels were raised on each acre if 2380 bushels of grain were raised on 68 acres?
4. Find the cost of 58 pounds of butter at $0.38 a pound.
5. Add: seventy-five, three hundred thirty-nine, two hundred five, eighteen, three thousand eighty-one.
6. A square field is 76 feet on a side. How many feet around the field?
7. A farmer has 708 trees in an orchard. $\frac{3}{4}$ of them are bearing fruit. How many of them are bearing?
8. A railroad train goes 45 miles an hour. How far does it go in 36 hours?

9. Find the cost of 15 pounds of sugar at 6 cents a pound, and ¼ pound of tea at 60 cents a pound.
10. How many feet in 25 miles?
11. At $7 a ton, how many tons of coal can be bought for $602?
12. How many years between the year DCCCLXIII and the year M?
13. How many hours in a year?
14. If a barrel of flour holds 196 pounds, how much does ¾ of a barrel hold?
15. What is the difference between 10000 and 7068?
16. In a class of 48 pupils, 25% failed. How many failed?
17. Add $27.58, $34.05, $32.41, $18, $54.50 and $0.75.
18. How many pencils in 15½ dozens?
19. A boy sold 3 bushels of peanuts at 5 cents a quart. What did he get for them?
20. What number must be added to 397 to make 750?
21. How many days in ⅗ of a year?
22. Write in words:—310005; 407327; 2000018.
23. A room is 48 feet long. How many feet wide must it be to contain 1728 square feet?
24. A man receives a salary of $1500 a year. What is his monthly salary?
25. There are 5 dozen lemons in one box and 53 lemons in another. How many lemons are in both boxes?

1.	2	3	4	5	6	7	8	9	10	11	12
2.	4	6	8	10	12	14	16	18	20	22	24
3.	6	9	12	15	18	21	24	27	30	33	36
4.	8	12	16	20	24	28	32	36	40	44	48
5.	10	15	20	25	30	35	40	45	50	55	60
6.	12	18	24	30	36	42	48	54	60	66	72
7.	14	21	28	35	42	49	56	63	70	77	84
8.	16	24	32	40	48	56	64	72	80	88	96
9.	18	27	36	45	54	63	72	81	90	99	108
10.	20	30	40	50	60	70	80	90	100	110	120
11.	22	33	44	55	66	77	88	99	110	121	132
12.	24	36	48	60	72	84	96	108	120	132	144

	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	
2	3	4	5	6	7	8	9		
3	4	5	6	7	8	9			
4	5	6	7	8	9				
5	6	7	8	9					
6	7	8	9						
7	8	9							
8	9								
9									

FOURTH GRADE FIRST HALF YEAR

LESSON I

Review

1 Check results after adding.

1.	2.	3.	4.	5.	6.	7.
873	725	892	690	173	123	135
+924	+109	+375	+375	+369	+456	+790
367	867	764	724	246	789	986
609	543	609	387	135	987	423
138	628	813	456	357	654	697
646	379	767	683	468	321	532
795	286	504	715	963	760	835
352	938	267	869	864	852	276
627	476	392	184	642	546	364

2 Find the sum:

1. Of 1089, 63, 794, 807, 98, 675, 2871, 487, 565.
2. Of 250, 376, 85, 1964, 555, 867, 623, 3698, 73.
3. Of 198, 77, 609, 875, 2567, 666, 3187, 370, 963.

3 Check results:

1.	2.	3.	4.	5.
19864	100000	369725	9073	37098
−16375	−65893	−107851	−6524	−8735

6. 1000000	7. 762304	8. 16750	9. 6750	10. 83246
−873508	−463758	−9087	−2345	−7850

4 Check results:

1. 978	2. 703	3. 653	4. 6943	5. 3524	6. 2598
×365	×237	×705	×26	×508	×609

5 Multiply:
1. 521 by 15. Multiply the product by 30
2. 693 by 850. " " " by 20
3. 7608 by 596. " " " by 10

6 Check results:

1. Divide 6907 by 8; by 5; by 3.
2. " 1500 by 9; by 4; by 6.

3. 16128 ÷ 64 4. 32805 ÷ 27 5. 362108 ÷ 512
6. 115844 ÷ 86 7. 43258 ÷ 86 8. 44204 ÷ 73
9. 7083 ÷ 352 10. 54648 ÷ 759

7 Read (or write in words):

657	75000	100736	888888
3280	98	39085	1111
908	705	2657903	3065065
3932	10600	1055	69402

8 Write in figures:
1. Eleven thousand.
2. One million, one hundred thousand, one hundred.
3. Five hundred fifty-five.

4. Five hundred thousand fifty.
5. Six million, eight thousand, three hundred three.
6. Four hundred sixty-three thousand, nine hundred nine.
7. Ten thousand, one hundred one.
8. One hundred ten thousand, one hundred ten.
9. Five thousand five.
10. Six thousand fifty-five.

Read and write in figures:

MDCCC; XCIII; CIII; MCDXCII; DXLV; CXIX; MDCCLXXVI; CMCXIX; LVIII; LXXXVIII.

Change to Roman numerals:

57; 46; 93; 19; 33; 125; 312; 1540; 12; 24; 1915; 1444.

LESSON II

Definitions of Terms Previously Used

9 ADDITION is the process of uniting several numbers into one number.

The numbers to be added are the ADDENDS.

The result in addition is called the SUM.

The *addends* and the *sum* are called the TERMS of addition.

The plus (+) sign is used to indicate addition.

+64 addend
43 addend
96 addend
203 sum

Find the results of the following, and name each addend.

(1) +6987	(2) +4987	(3) +2986	(4) +9847	(5) +9877
4329	9876	4987	7777	8555
9876	9875	6914	8888	6698
4329	5498	9876	9198	7149
8769	8765	7789	7649	8694
4329	4329	9876	4987	7659
	4368	9875	6149	4187

Find the sum if the addends are:

(6) 14986, 42907, 89145, 219876, 419, 614, 9876.

(7) 299, 9876, 14098, 4168, 7698, 98764, 49098.
(8) 917, 9187, 9976, 6849, 61498, 41689, 4137.
(9) XL, MCLXVI, CCCLXVIII, CCXCVII, LIV, DLXXIX.
(10) 986, 4976, 12345, 9814, 298, 4713, 98643.

10 SUBTRACTION is the process of taking one number from another.

The number from which another number is taken is called the MINUEND.

The number taken from the minuend is called the SUBTRAHEND.

The result in subtraction is called the REMAINDER or DIFFERENCE.

The *minuend, subtrahend, remainder* or *difference* are called the TERMS of subtraction.

The minus (−) sign indicates subtraction.

$$\begin{array}{rl} 4962 & \text{minuend} \\ 1987 & \text{subtrahend} \\ \hline 2975 & \text{remainder or difference.} \end{array}$$

1. If the minuend is 11111, find the successive remainders if the subtrahends are:
   9167, 4139, 987, 4136, 9085, 2169, 10095, 98.
2. If the subtrahend is 98765, find the remainders if the minuends are:
   100014, 216019, 98989, 123456, 614203, 912691.

11 MULTIPLICATION is the process of taking one number as many times as there are *ones* or *units* in another. The number to be taken or multiplied is called the MULTIPLICAND.

The number showing how many times the multiplicand is taken is called the MULTIPLIER.

The result in multiplication is called the PRODUCT.

The *multiplicand* and *multiplier* are called FACTORS of their *product*.

The *multiplicand*, *multiplier*, and *product* are called the TERMS of multiplication.

The sign (×) indicates multiplication.

$$\begin{array}{rl} 25 & \text{multiplicand} \\ 15 & \text{multiplier} \\ \hline 125 & \\ 25 & \\ \hline 375 & \text{product} \end{array}$$

Multiplicands: 9871; 14598; 2987; 4096; 986; 497.
Multiplicers: 298; 496; 907; 419; 8164; 29; 98.
Find products.

12 DIVISION is the process of finding how many times one number is contained in another, or of separating a number into equal parts.

The number to be divided into another number is called the DIVISOR.

The number to be divided by another number is called the DIVIDEND.

The result in division is called the Quotient.

Sometimes the division will not be even; then there is a Remainder.

The *divisor*, *dividend*, *quotient*, and *remainder* (if any) are called the Terms of division.

The signs (÷) ()____), and (——) indicate division.

$$\begin{array}{rrl}
 & 701 & \text{quotient} \\
\text{divisor } 13) & \overline{9117} & \text{dividend} \\
 & \underline{91} & \\
 & 17 & \\
 & \underline{13} & \\
 & 4 & \text{remainder}
\end{array}$$

Dividends: 109876; 2198714; 219870; 300000; 475125; 90000.

Divisors: 214; 1025; 196; 487; 375; 1987; 681.

Find quotients and remainders. (Check results.)

13 Notation is the process of writing numbers.

Numeration is the process of reading numbers.

Write in figures:

(1) One million, six hundred, seven.
(2) Three hundred, seven thousand, eighty-four.
(3) Seventy-six thousand, ninety.
(4) Eight million, forty-nine.
(5) Seventy thousand, one.
(6) Seventeen thousand, six hundred, ten.
(7) Nineteen hundred, ninety-three.
(8) Six hundred ten thousand, six hundred, one.

Read or write in words:

(9) 1234567; (10) 90012; (11) 219876;
(12) 2001002; (13) 416872; (14) 2146800;
(15) 416890; (16) 123461; (17) 90104;
(18) 2170061; (19) 20105.

NOTE.—Pupils should prove or check each problem, and so ensure accurate results. They should also be taught to be prompt in attaining results. *Efficiency, accuracy, promptness* is a good motto.

14 See if you can work these groups of problems correctly in the time limit set. Your teacher will call the time. Be very accurate as well as prompt, and so prove your efficiency.

(1) 5 minutes. Add 64, 96, 75, 43, 74, 61.

$$\begin{array}{r} 49161 \\ -29874 \\ \hline \end{array} \qquad \begin{array}{r} 498 \\ \times 9 \\ \hline \end{array} \qquad \begin{array}{r} 296 \\ \times 8 \\ \hline \end{array} \qquad \begin{array}{r} 49 \\ \times 43 \\ \hline \end{array}$$

(2) 3 minutes.

$$\begin{array}{r} +986 \\ 419 \\ 816 \\ 432 \\ 987 \\ \hline \end{array} \qquad \begin{array}{r} +219 \\ 876 \\ 413 \\ 217 \\ 219 \\ \hline \end{array} \qquad \begin{array}{r} +98 \\ 75 \\ 43 \\ 96 \\ 75 \\ \hline \end{array}$$

(3) 5 minutes.

$$\begin{array}{r} 416 \\ \times 217 \\ \hline \end{array} \qquad \begin{array}{r} 986 \\ \times 125 \\ \hline \end{array}$$

(4) 1 minute.

$$\begin{array}{r} 2189 \\ \times 25 \\ \hline \end{array}$$

(5) 1 minute.

$$\begin{array}{r} 689 \\ \times 76 \\ \hline \end{array}$$

(6) 3 minutes. 127)901635

(7) 3 minutes. 298)129876

(8) 5 minutes. 75)90753

(9) 5 minutes. 29)1234567

Prove.

(10) 3 minutes. 9)123716

4)1002171

$$\begin{array}{r}468\\ \times 25\\ \hline\end{array}$$

3)97645

LESSON III

Factors

15 7 times 3 = ? 21 is the product of what numbers? Then 7 and 3 are *factors* of 21.

2 times 3 times 5 = ? 30 is the product of what numbers? Then 2, 3, and 5 are factors of 30.

Definition:—The numbers multiplied together to give a product are FACTORS of that product.

(1) Give two factors of 6, 9, 10, 14, 15, 25, 35, 49, 72, 100,
(2) Give three factors of 8, 12, 18, 24, 30, 32, 36, 40, 50, 75.

(3)

	2		2		2
	3		3		4
24 =	4 times	36 =	4 times	40 =	5 times
	6 what?		6 what?		8 what?
	8		9		10
	12		12		20

Give all the different factors of 24; 36; 40; 12; 16; 30.

(4)
The factors of 72 are 9 and —; 24 and —; 2, 4, and —
100 are 10 and —; 20 and —; 5, 10, and —
36 are 4 and —; 6 and —; 3, 3, and —

16 $2 = 2$
$3 = 3$
$5 = 5$
$7 = 7$
$11 = 11$
$13 = 13$
times what?

What are the factors of 2, 3, 5, 7, 11, 13?
What other factors has 2?
What other factors has 3, 5, 7, 11, 13?

Definition:—A number that has no other factors besides itself and one is a *Prime Number*.

$4 = 4$
$6 = 6$
$12 = 12$
$12 = 6$
$12 = 4$
times what?

What other factors has 4?
What other factors has 6?
What other factors, besides itself and 1, has each of the following: 8, 9, 20, 30, 36, 48?

17 Definition:—A number that has other factors, besides itself and one, is a *Composite Number*.

(1) Tell whether each of the following is a prime or composite number; give the reason: 9, 5, 8, 20, 11, 31, 32, 19, 3, 45, 30, 13, 2, 41, 50, 29, 16, 17, 7, 23, 60, 15, 22.

(2) Find all the prime numbers from 1 to 100.

(3) Find all the composite numbers to 100.

The factors of		
4	are	2 and 2.
6		2 and 3.
9		3 and 3.
10		5 and 2.
14		7 and 2.
15		5 and 3.
21		7 and 3.
30		2, 3, and 5.
12		2, 2, and 3.
18		2, 3, and 3.

Are these factors prime or composite numbers?

18 Definition.—A factor which is a *prime number* is a *Prime Factor.*

(1) Give 2 prime factors of 22, 25, 33, 35, 49, 55, 26.
(2) Give 3 prime factors of 42, 28, 56, 99, 45, 70, 20.

(3) $7 \times 3 = ?$ 7 is contained how many times in 21?
3 is contained how many times in 21?
What are the factors of 21?
What numbers will divide 21 without a remainder?

(4) $2 \times 3 \times 5 = ?$ What are the prime factors of 30?
What numbers will divide 30 without a remainder?

19 Definition:—A *factor* of a number is also a *divisor* of the number.

What are the prime factors of 180?

Since 180 is an even number (even numbers end with 0, 2, 4, 6, or 8), it can be divided by 2 without remainder. It has the factors 2, a prime number, and 90, a composite number. Since 90 is an even number, it can be divided by 2; it has the factors 2, a prime number and 45, a composite number. Since 45 is an odd number (odd numbers end in 1, 3, 5, 7, or 9), it can not be divided *exactly* by 2. It can be divided by 3;

Written
Process.

$$\begin{array}{r|r}
2 & 180 \\ \hline
2 & 90 \\ \hline
3 & 45 \\ \hline
3 & 15 \\ \hline
 & 5
\end{array}$$

2, 2, 3, 3, 5
Prime factors of 180.

its factors are 3, prime, and 15, composite. 15 can be divided by 3; its factors are 3 and 5, both prime numbers. Hence, 180 can be divided successively by the prime numbers 2, 2, 3, 3, 5; and 180 is the product of 2, 2, 3, 3, and 5.

The prime factors of 180 are 2, 2, 3, 3, and 5.

Proof-check: $2 \times 2 \times 3 \times 3 \times 5 = 180$

(1) Find the prime factors of the following: (1) by separating the numbers into factors, (2) by division: 24, 30, 36, 40, 45, 50, 64, 70.

(2) Find the prime factors of the following by division:

Form:

$$\begin{array}{r|r} 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

96	125	400	315
84	120	184	360
231	115	140	100
144	180	273	231
390	330	222	164

2, 3, 5, 7 = Answer $2 \times 3 \times 5 \times 7 = 210$, Proof-check

20 Say the multiplication table by 2. Every product in the table can be divided by 2 without remainder.

Beginning at 0, count by 2's to 50. Every even number can be divided by 2 without remainder.

Say the multiplication table by 5. Every product can be divided by 5 without remainder.

Beginning at 0, count by 5's to 50.

Every number ending in 0 or 5 can be divided by 5 without remainder.

Definitions:—A number that will divide another number without remainder is an *Exact Divisor* of that number.

A number that can be exactly divided by 2 is an *Even Number.*

A number that can not be exactly divided by 2 is an *Odd Number.*

(1) Write the odd numbers to 100.
(2) Write the even numbers to 100.
(3) Write all the exact divisors of 72. Which of the divisors are prime?

(4) Write all the exact divisors of
8
12
15
18
20
24
25
30
Tell which divisors are prime numbers.

** ** ** ** | ** ** ** **
21 * * * * | * * * *

How many times are 4 times 3 contained in 8 times 3? This may be written thus:

8 times 3 ÷ 4 times 3 = 2.

$$\frac{\overset{2}{\cancel{8}}\times\cancel{3}}{\cancel{4}\times\cancel{3}}=2$$

The dividend, above the line, contains the factors 8 and 3. The divisor, below the line, contains the factors 4 and 3. 3 is a factor of both dividend and divisor. 3 is called a common factor. We can take away, or cancel, the common factor from dividend and divisor, and see that 4 is contained in 8, 2 times.

Definition:—The short way of dividing, by taking away common factors from dividend and divisor, is called *Cancellation.*

1. Find the quotients by cancellation:

$$\frac{2\times7\times5\times11}{7\times11} \qquad \frac{17\times8\times5\times6\times7}{7\times6\times17}$$

$$\frac{3\times8\times9\times5\times2}{8\times5\times3} \qquad \frac{4\times23\times5\times13\times3}{13\times4\times23}$$

2. Divide by separating into prime factors and cancelling:

$$\frac{2310}{210}=\frac{\cancel{2}\times\cancel{3}\times\cancel{5}\times\cancel{7}\times11}{\cancel{2}\times\cancel{3}\times\cancel{5}\times\cancel{7}}=11,\text{ Quotient}$$

$$\frac{546}{39} \qquad \frac{765}{51} \qquad \frac{968}{44} \qquad \frac{462}{42}$$

3. Divide the product of 5 times 23 times 17 by 23 times 5.

4. Divide: $6\times5\times11\times3\times7$ by $11\times7\times5$
$13\times2\times17\times43\times5$ by $43\times17\times13$

5. Divide:

$$\frac{\overset{\cancel{3}}{\cancel{6}}\times9\times\overset{3}{\cancel{15}}\times\overset{\cancel{2}}{\cancel{20}}}{\underset{\cancel{3}}{\cancel{30}}\times\cancel{5}\times\underset{\cancel{2}}{\cancel{4}}\times7}=\frac{3\times9}{7}=\frac{27}{7}=3\tfrac{6}{7}\text{ Ans.}$$

20 in the dividend = 10 × 2 30 in the divisor = 10 × 3	Take away the common factor 10. Write the other factor of 20 (2) in the dividend, and the other factor of 30 (3) in the divisor.
15 in the dividend = 5 × 3 5 in the divisor = 5 × 1	Take away the common factor 5. Write 3 in the dividend. We do not write the factor 1.
6 in the dividend = 3 × 2 4 in the divisor = 2 × 2	Take away the common factor 2. Write 3 in the dividend; 2 in the divisor.

Cancel the common factors now seen in the dividend and divisor. There remain in the dividend the factors 9 and 3; in the divisor, 7.

6.
$$\frac{6\times12\times15\times36}{5\times4\times3\times24}$$

7.
$$\frac{45\times30\times18}{25\times9\times5}$$

8.
$$\frac{25\times2\times14\times36}{48\times30\times7}$$

9.
$$\frac{5\times8\times30\times24}{6\times8\times4}$$

10.
$$\frac{2\times5\times11\times9\times14}{4\times6\times7\times10}$$

11.
$$\frac{12\times6\times20\times5}{3\times60\times4}$$

22 Applications: If 8 tons of coal cost \$46.40, what will 12 tons of the same kind cost? 1 ton will cost $\frac{1}{8}$ of \$46.40 or 8 divided into \$46.40, or $\frac{46.40}{8}$. 12 tons will cost 12 times as much as 1 ton.

$$\frac{\overset{23.20}{\cancel{\$46.40}}\times\overset{3}{\cancel{12}}}{\underset{\cancel{2}}{\cancel{8}}}=23.20\times 3=\$69.60.\quad\text{Answer.}$$

(1) If 24 barrels of flour cost \$180.00, what will 15 barrels of the same kind cost?

(2) How many pounds of sugar at 6 cents a pound must be given for 9 dozen eggs at 23 cents a dozen?

(3) How many tiles 4 inches by 3 inches will be needed to cover a space 10 feet by 12 feet? 10×12 inches? 12×12 inches?

(4) How many baskets of grapes, each holding 10 quarts, at 6 cents a quart will pay for 15 pounds of butter at 28 cents a pound?

(5) If 25 cheeses cost \$12.50, what will 40 of the same kind cost?

23

$$50=2\times5\times5$$
$$70=2\times5\times7$$

What prime factors are common to 50 and 70? 2 and 5 are common factors of 50 and 70. Therefore, this product of 2 and 5 or 10 is a common factor of 50 and 70. What is the highest, or largest, common factor of 50 and 70?

We can find the largest common factor of 2 numbers by division.

50)70(1 50	50 divided into 70, 1 time, 20 remainder.
20)50(2 40	20 divided into 50, 2 times, 10 remainder.
10)20(2 20	10 divided into 20, 2 times, 0 remainder.

The highest common factor of 50 and 70 is 10.

The greatest common divisor of 50 and 70 is 10.

Definition:—The largest number that is a factor, or an exact divisor, of two numbers, is called the *Highest Common Factor*.

Find the highest common factor of the following:

1. 33, 154;
2. 72, 108;
3. 135, 225;
4. 144,180;
5. 123, 385;
6. 60, 270;
7. 64, 88;
8. 245, 343;
9. 84, 189;
10. 81, 189;
11. 231, 363;
12. 70, 168.

24

5×4 = 20	20 is the product of 5 and 4. It contains 4 five times.
7×4 = 28	28 is the product of 7 and 4. It contains 4 seven times.

20 is a *multiple* of 5 and 4.

28 is a multiple of 7 and 4.

6×4 = 24	24 is a multiple of 6 and 4.
8×3 = 24	24 is a multiple of 8 and 3.

24 is a common multiple of 6, 4, 8, and 3.

4 is exactly contained in 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48.
6 is exactly contained in 12, 18, 24, 30, 36, 42, 48, 54.
8 is exactly contained in 16, 24, 32, 40, 48, 56, 64, 72.
What numbers contain 4, 6, and 8 an exact number of times? Of these, which is the smallest?

Definition:—The smallest number that exactly contains each of two or more numbers is called their *Lowest Common Multiple.*

We can find the lowest common multiple of 4, 6, and 8 by factoring.

$4 = 2 \times 2$
$6 = 3 \times 2$
$8 = 2 \times 2 \times 2$
$2 \times 2 \times 2 \times 3 = 24$

Their lowest common multiple must contain the factor 2 three times, since $8 = 2 \times 2 \times 2$; also, the factor 3, since $6 = 3 \times 2$.

We can also find it by dividing by prime factors of 4, 6, and 8.

$$\begin{array}{r|l} 2 & 4 - 6 - 8 \\ \hline 2 & 2 - 3 - 4 \\ \hline & 1 - 3 - 2 \end{array}$$

$2 \times 2 \times 3 \times 2 = 24$

Divide by the common factor 2.

Divide again by the common factor 2, bringing down 3, which does not contain that factor.

3 and 2 have no common factor.

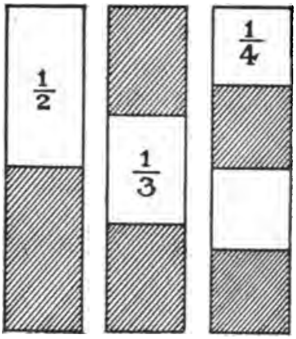
The lowest common multiple is the product of the divisors and the last quotients.

Find the lowest common multiple by dividing:

1. 6, 10, 12;
2. 18, 24, 36;
3. 16, 24, 8;
4. 10, 24, 12;
5. 45, 60, 36;
6. 20, 16, 40;
7. 15, 18;
8. 72, 48;
9. 105, 120;
10. 25, 10, 50;
11. 20, 50, 25;
12. 15, 24, 40.

LESSON IV

Fractions



The first oblong is divided into two equal parts. Each part is called one-half or $\frac{1}{2}$.

The second oblong is divided into three equal parts. Each part is called one-third or $\frac{1}{3}$.

The third oblong is divided into four equal parts. Each part is called one-fourth or $\frac{1}{4}$.

$\frac{1}{2}$ is a part or *fraction* of a whole oblong. How many halves = 1?

$\frac{1}{3}$ is a part or *fraction* of a whole oblong. How many thirds = 1?

$\frac{1}{4}$ is a part or *fraction* of a whole oblong. How many fourths = 1?

A *fraction* is one or more of the equal parts into which a unit may be divided; as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{8}$.

A fraction has two parts: a number above a line, and a number below the line.

The number below the line is called the DENOMINATOR, and indicates into how many equal parts a unit has been divided. In $\frac{3}{8}$, a unit has been divided into 8 equal parts.

The number above the line is called the NUMERATOR, and indicates how many of the equal parts are taken. In $\frac{3}{8}$, a unit has been divided into 8 equal parts, and 3 of them are taken.

The *Numerator* and *Denominator* are called the TERMS of the fraction.

A fraction also indicates division. $\frac{1}{8}$ means that one has been divided by eight.

The numerator is the dividend; the denominator is the divisor. $\frac{1}{8}$ means $1 \div 8$; $\frac{2}{8}$ means $2 \div 8$.

1. Read: $\frac{1}{4}$, $\frac{1}{5}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{7}{8}$, $\frac{7}{10}$, $\frac{9}{10}$, $\frac{5}{8}$, $\frac{3}{8}$, $\frac{14}{15}$, $\frac{11}{12}$, $\frac{1}{10}$, $\frac{9}{11}$.

2. Write: nine-tenths; one-seventh; three-fifths; one-tenth; seven-twelfths; one-half.

Make diagrams showing:

3. One-fifth; three-fourths; five-eighths; two-thirds; eight-ninths; two-fifths.

In one whole (or unit) are how many halves, fourths, sevenths, thirds, sixths, twentieths, twelfths?

4. Write the fraction in which
the denominator is 7, the numerator is 3
" 8, " 7
" 15, " 11
" 25, " 21
" 16, " 9

5. The numerator is 9, the denominator is 10
" 8, " 9
" 7, " 10
" 1, " 2
" 3, " 5
" 4, " 9

6. Read and write the names of

$\frac{9}{15}$, $\frac{7}{12}$, $\frac{19}{20}$, $\frac{1}{9}$, $\frac{16}{12}$, $\frac{4}{7}$, $\frac{20}{25}$, $\frac{19}{30}$, $\frac{16}{17}$, $\frac{4}{23}$.

7. $\frac{8}{8}$ = how many whole ones (or units)?

$\frac{16}{8}$ = how many whole ones?

8. $\frac{9}{8}$ = how many whole ones and how many eighths besides?

$\frac{8}{8}=1$, $\frac{16}{8}=2$; $\frac{9}{8}=1$ and $\frac{1}{8}$ besides.

A fraction which equals a unit or more than a unit is called an IMPROPER FRACTION; that is, when the numerator is equal to or more than the denominator; as $\frac{9}{8}$, $\frac{4}{3}$, $\frac{3}{2}$, $\frac{9}{9}$.

A fraction is called a proper fraction when the numerator is less than the denominator; as $\frac{2}{3}$, $\frac{1}{7}$, $\frac{6}{11}$, $\frac{4}{9}$, $\frac{19}{20}$.

An expression is called a Mixed Number when it is composed of an integer and a fraction; as $2\frac{1}{2}$, $1\frac{1}{3}$, $7\frac{2}{5}$, $6\frac{4}{9}$.

$\frac{4}{3}$ = how many units?

$1 = \frac{3}{3}$. Therefore in $\frac{4}{3}$ there will be 1 unit and $\frac{1}{3}$ over.

$\frac{4}{3} = 1\frac{1}{3}$.

$\frac{11}{3}$ = how many units? Every time there are $\frac{3}{3}$ there will be one unit; therefore $\frac{11}{3} = 3$ units and $\frac{2}{3}$ over. $\frac{11}{3} = 3\frac{2}{3}$.

Hence, every improper fraction may be changed to an integer or a mixed number.

Rule: Divide the numerator by the denominator to obtain the integer. Set the remainder, if any, over the denominator to obtain the fractional part.

Oral or Written Work

Change to integers or mixed numbers:

1. $\frac{2}{2}$, $\frac{3}{2}$, $\frac{7}{2}$, $\frac{4}{3}$, $\frac{7}{3}$, $\frac{12}{3}$, $\frac{14}{3}$, $\frac{7}{4}$, $\frac{11}{4}$, $\frac{19}{5}$, $\frac{17}{8}$, $\frac{19}{3}$, $\frac{21}{5}$.
2. $\frac{43}{8}$, $\frac{17}{4}$, $\frac{27}{7}$, $\frac{29}{3}$, $\frac{30}{6}$, $\frac{30}{7}$, $\frac{30}{4}$, $\frac{30}{5}$, $\frac{31}{4}$, $\frac{43}{5}$, $\frac{39}{8}$.
3. $\frac{27}{12}$, $\frac{31}{15}$, $\frac{43}{17}$, $\frac{50}{7}$, $\frac{50}{12}$, $\frac{50}{15}$, $\frac{43}{20}$, $\frac{35}{6}$, $\frac{39}{7}$, $\frac{49}{9}$.

26 *Analysis:*

$2\frac{1}{4}$ = how many fourths?
$1 = \frac{4}{4}$.
$2 = \frac{8}{4}$; and $\frac{1}{4}$ more will equal $\frac{9}{4}$.

Analysis:

$7\frac{2}{3}$ = how many thirds?
$1 = \frac{3}{3}$.
$7 = \frac{21}{3}$; and $\frac{2}{3}$ more will equal $\frac{23}{3}$.

Hence, every mixed number may be changed to an improper fraction.

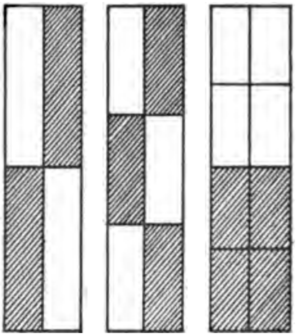
Rule: Multiply the integral part by the denominator of the fraction, and add to this product the numerator; set this result over the denominator; thus $9\frac{1}{2} = \frac{19}{2}$; $3\frac{4}{7} = \frac{25}{7}$.

Oral or Written Exercises

Change to improper fractions:

1. $2\frac{1}{2}$, $7\frac{2}{3}$, $9\frac{4}{5}$, $11\frac{1}{4}$, $3\frac{5}{9}$, $6\frac{2}{5}$, $7\frac{1}{4}$, $19\frac{2}{3}$, $17\frac{3}{4}$, $4\frac{9}{10}$.
2. $3\frac{7}{8}$, $3\frac{2}{11}$, $7\frac{1}{3}$, $14\frac{2}{3}$, $7\frac{5}{8}$, $3\frac{7}{9}$, $2\frac{1}{5}$, $9\frac{2}{3}$, $3\frac{7}{10}$, $4\frac{1}{2}$.
3. $15\frac{3}{6}$, $100\frac{2}{3}$, $99\frac{1}{2}$, $17\frac{2}{5}$, $50\frac{1}{3}$, $13\frac{4}{5}$, $63\frac{1}{4}$, $27\frac{4}{9}$, $16\frac{5}{8}$, $20\frac{7}{9}$.
4. $50\frac{3}{7}$, $19\frac{7}{15}$, $25\frac{3}{5}$, $12\frac{9}{12}$, $40\frac{2}{7}$, $50\frac{7}{12}$, $52\frac{2}{3}$, $67\frac{1}{9}$, $43\frac{7}{10}$, $19\frac{7}{15}$.

27



The first oblong is divided into how many equal parts? Each part is called $\frac{1}{4}$.

Each half contains how many fourths? $\frac{1}{2} = \frac{2}{4}$.

In the second oblong each half equals how many sixths? $\frac{1}{2} = \frac{3}{6}$.

In the third oblong each half equals how many eighths? $\frac{1}{2} = \frac{4}{8}$.

$$\frac{1\times4}{2\times4} = \frac{4}{8}$$

$$\frac{4\div4}{8\div4} = \frac{1}{2}$$

A fraction may be changed to a higher or lower denominator, if the numerator is changed in the same way at the same time; for this is really, as you can see, only multiplying or dividing by 1 ($\frac{4}{4}$, $\frac{3}{3}$, etc.), which does not change the value, only the form of the fraction.

Changing a fraction, in this way, to an equivalent fraction is called Reduction to a higher denominator, or Reduction to a lower denominator.

Change $\frac{4}{5}$ to twentieths. $\frac{4}{5} \times \frac{4}{4} = \frac{16}{20}$

Change $\frac{9}{12}$ to fourths. $\frac{9}{12} \div \frac{3}{3} = \frac{3}{4}$.

Rule: To reduce a fraction to a higher denominator, divide the required denominator by the given denominator; then multiply both terms of the given fraction by this quotient.

To reduce a fraction to a lower denominator, divide the given denominator by the required denominator; then divide both terms of the given fraction by this quotient.

Oral and written:

Change:

1. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{6}$, $\frac{2}{6}$, $\frac{5}{6}$, $\frac{3}{6}$, to twelfths.
2. $\frac{1}{3}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{3}{6}$, $\frac{5}{6}$, $\frac{4}{6}$, to 30ths.
3. $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$, $\frac{9}{14}$, $\frac{3}{14}$, $\frac{11}{14}$, to 28ths.
4. $\frac{3}{20}$, $\frac{7}{20}$, $\frac{5}{8}$, $\frac{7}{10}$, $\frac{4}{5}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{13}{20}$, to 40ths.

5. $$\left.\begin{array}{c}\frac{5}{8}\\ \frac{7}{16}\\ \frac{13}{16}\\ \frac{3}{4}\end{array}\right| = \frac{?}{32}$$

6. $$\left.\begin{array}{c}\frac{1}{3}\\ \frac{5}{6}\\ \frac{7}{8}\\ \frac{3}{4}\\ \frac{1}{12}\end{array}\right| = \frac{?}{24}$$

7. $$\left.\begin{array}{c}\frac{1}{5}\\ \frac{7}{10}\\ \frac{24}{25}\\ \frac{1}{2}\\ \frac{4}{5}\end{array}\right| = \frac{?}{50}$$

8. $$\left.\begin{array}{c}\frac{1}{2}\\ \frac{2}{3}\\ \frac{3}{4}\\ \frac{4}{5}\\ \frac{5}{6}\end{array}\right| = \frac{?}{60}$$

9. $$\left.\begin{array}{c}\frac{9}{12}\\ \frac{12}{16}\\ \frac{15}{20}\\ \frac{12}{24}\\ \frac{30}{40}\end{array}\right| = \frac{?}{4}$$

10. $$\left.\begin{array}{c}\frac{20}{30}\\ \frac{18}{27}\\ \frac{16}{24}\\ \frac{24}{36}\\ \frac{16}{48}\end{array}\right| = \frac{?}{3}$$

11. $\frac{21}{28} = \frac{?}{7}$
$\frac{11}{44} = \frac{}{4}$
$\frac{15}{60} = \frac{?}{20}$
$\frac{42}{56} = \frac{?}{8}$

12. $\frac{25}{30} = \frac{?}{6}$
$\frac{42}{48} = \frac{?}{8}$
$\frac{14}{49} = \frac{?}{7}$
$\frac{20}{32} = \frac{?}{8}$

28 A fraction is changed to its Lowest terms by dividing both terms by the highest common factor of the numerator and denominator.

Change or reduce $\frac{9}{12}$ to lowest terms.

$9 = 3 \times 3$
$12 = 3 \times 2 \times 2$
$3 = \text{H. C. F.}$

$$\frac{9 \div 3}{12 \div 3} = \tfrac{3}{4} \text{ lowest terms.}$$

Change $\frac{32}{48}$ to lowest terms.

$32 = 2 \times 2 \times 2 \times 2 \times 2$
$48 = 2 \times 2 \times 2 \times 2 \times 3$
$2 \times 2 \times 2 \times 2 = 16$
$16 = \text{H. C. F.}$

$$\frac{32 \div 16}{48 \div 16} = \tfrac{2}{3} \text{ lowest terms.}$$

Reduce to lowest terms:

1. $\frac{9}{27}$ 2. $\frac{100}{125}$ 3. $\frac{14}{42}$ 4. $\frac{19}{57}$ 5. $\frac{33}{55}$ 6. $\frac{16}{24}$
7. $\frac{42}{60}$ 8. $\frac{36}{54}$ 9. $\frac{45}{60}$ 10. $\frac{34}{51}$ 11. $\frac{30}{48}$ 12. $\frac{18}{24}$
13. $\frac{24}{40}$ 14. $\frac{45}{55}$ 15. $\frac{40}{56}$ 16. $\frac{50}{75}$ 17. $\frac{35}{56}$ 18. $\frac{16}{36}$
19. $\frac{45}{50}$ 20. $\frac{24}{27}$ 21. $\frac{7}{28}$ 22. $\frac{15}{45}$ 23. $\frac{28}{56}$ 24. $\frac{17}{34}$
25. $\frac{24}{32}$ 26. $\frac{27}{36}$ 27. $\frac{28}{35}$ 28. $\frac{36}{60}$ 29. $\frac{24}{42}$ 30. $\frac{12}{48}$

Common Denominator

29 Change $\frac{1}{2}$ and $\frac{1}{6}$ to the same or a common denominator. That denominator may be any number into which 2 and 6 will both divide, as 12, 24, 48, 60, 120. It is best to select the least number—that is, the lowest common multiple of 2 and 6.

$2 = 2$
$6 = 2 \times 3$
$2 \times 3 = 6$
$6 = \text{L. C. M.}$

$$\left.\begin{array}{l} \dfrac{1 \times 3}{2 \times 3} = \tfrac{3}{6} \\[2ex] \dfrac{1 \times 1}{6 \times 1} = \tfrac{1}{6} \end{array}\right| \text{ L. C. D.}$$

Reduce $\frac{3}{4}$ and $\frac{5}{6}$ to the lowest common denominator.

$4 = 2 \times 2$
$6 = 2 \times 3$
$2 \times 2 \times 3 = 12$
$12 =$ L. C. M.

$$\left.\begin{array}{l}\dfrac{3\times 3}{4\times 3}=\frac{9}{12}\\[2ex] \dfrac{5\times 2}{6\times 2}=\frac{10}{12}\end{array}\right| \text{ L. C. D.}$$

Rule: To reduce several fractions to the lowest common denominator, find the lowest common multiple of the denominators; then change each fraction to an equivalent fraction having as a denominator this common multiple.

Reduce to lowest common denominator:

1. $\frac{1}{3}$, $\frac{1}{2}$; 2. $\frac{4}{5}$, $\frac{7}{15}$; 3. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$; 4. $\frac{15}{32}$, $\frac{15}{16}$, $\frac{7}{8}$;
5. $\frac{3}{4}$, $\frac{2}{3}$; 6. $\frac{2}{3}$, $\frac{7}{8}$; 7. $\frac{2}{3}$, $\frac{7}{8}$, $\frac{5}{6}$; 8. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{16}$;
9. $\frac{7}{8}$, $\frac{5}{6}$; 10. $\frac{2}{3}$, $\frac{5}{21}$; 11. $\frac{1}{2}$, $\frac{7}{8}$, $\frac{2}{3}$; 12. $\frac{7}{10}$, $\frac{3}{4}$, $\frac{7}{8}$;
13. $\frac{7}{8}$, $\frac{11}{12}$; 14. $\frac{7}{8}$, $\frac{3}{20}$; 15. $\frac{9}{10}$, $\frac{14}{15}$, $\frac{4}{5}$, 16. $\frac{1}{2}$, $\frac{7}{8}$, $\frac{4}{5}$.

LESSON V

Review and Drill

30 Oral:

(1) What are addends? What is a sum?

(2) What is minuend? Subtrahend? Difference?

(3) What is multiplicand? Multiplier? Product?

(4) What is dividend? Divisor? Quotient?

(5) $9+8+3+5+6+3+9+7=?$

(6) $100-83$; $69-35$; $73-59$; $95-70$.

(7) 8×9; 80×9; 8×90; 100×9; 8×900.

(8) $15\div5$; $29\div5$; $35\div5$; $50\div5$; $75\div5$.

(9) $63+25$; $73+27$; $15+35$; $32+46$; $90+55$.

(10) Divide 7 into 63; 8 into 64; 6 into 72; 9 into 54.

31 Written:

(1) What is the sum of 246, 89, 327, 5604, 70, 698, 1564, 8293? Check.

(2) Find the product of 387 and 695.

(3) What is the quotient of 14245 divided by 35?

(4) How much greater than 659437 is 1000000?

(5) Add these results: $29351\div49$ and 426×79.

32 Oral:

(1) What is an even number? Say the even numbers to 20.

(2) What is an odd number? Say the odd numbers to 20.

(3) What are factors? Give factors of 12; 20; 50; 36.

(4) What factors are common to 16, 32, 40?

(5) What is the highest common factor of several numbers? What is the highest common factor of 16, 32, 40?

(6) What is a multiple of a number? What are multiples of 5?

(7) What is a common multiple of several numbers? Of 5, 10, 15?

(8) What is the lowest common multiple of several numbers? Of 5, 10, 15?

(9) What is the highest common factor of 20 and 30; 36 and 54; 32 and 40?

(10) What is the lowest common multiple of 4 and 6; 4, 6, and 8; 3 and 4; 3, 4, and 6; 2, 5, and 10; 3, 6, and 9; 15 and 20; 9 and 12?

33 Written:

(1) Find the highest common factor by division:

24 and 36;	50 and 80;	20 and 24;	36 and 48;
15 and 24;	30 and 42;	48 and 72;	25 and 45.

(2) Find the lowest common multiple by division:

6, 8, 12;	3, 4, 2;	5, 10, 15;	12, 15, 20;
15, 24;	30, 42;	48, 72;	25, 45.

(3) Divide, using cancellation:

$15 \times 12 \times 11$ by 11×5; $3 \times 14 \times 15 \times 15$ by 15×7.

(4) Separate into prime factors by division: 4620, 154. Divide 4620 by 154. Divide by cancellation.

(5) Write the prime numbers between 1 and 50.

34 Oral:

(1) What is a fraction? Give one.

(2) In the fraction $\frac{3}{5}$ name each term; tell what each means.

(3) What is a proper fraction? Give one.

(4) What is an improper fraction? Give one.

(5) What is a mixed number? Give one.

(6) $\left.\begin{matrix}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{4}\\ \frac{1}{6}\end{matrix}\right| =$ how many 12ths?

Have you changed the value of the fractions?

How have you changed them?

(7) Change to lowest terms:

$\frac{6}{12}, \frac{4}{16}, \frac{3}{9}, \frac{4}{20}, \frac{5}{15}, \frac{7}{14}, \frac{2}{12}, \frac{7}{21}.$

(8) What integer and what fraction do the following equal:

$\frac{16}{3}, \frac{15}{2}, \frac{18}{4}, \frac{27}{5}, \frac{64}{9}, \frac{50}{7}, \frac{29}{8}?$

(9) How many units in the following:

$\frac{12}{4}, \frac{18}{3}, \frac{20}{5}, \frac{18}{2}, \frac{24}{6}?$

(10) How many twelfths in 1, in 2, in $3\frac{1}{12}$, in $\frac{3}{4}$, in $4\frac{5}{12}$?

35 Written:

(1) Reduce to lowest terms:

$\frac{12}{18}$, $\frac{25}{35}$, $\frac{28}{49}$, $\frac{24}{56}$, $\frac{60}{72}$, $\frac{9}{12}$, $\frac{15}{20}$, $\frac{30}{36}$.

(2) Reduce to whole or mixed numbers:

$\frac{290}{24}$, $\frac{735}{45}$, $\frac{95}{18}$, $\frac{256}{16}$.

(3) Reduce to equivalent fractions having the lowest common denominator:

$\frac{2}{9}$, $\frac{1}{6}$, $\frac{2}{3}$; $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{2}$; $\frac{7}{10}$, $\frac{4}{5}$, $\frac{1}{6}$; $\frac{17}{20}$, $\frac{5}{8}$, $\frac{3}{10}$.

(4) Reduce to improper fractions:

$15\frac{3}{7}$, $8\frac{9}{10}$, $36\frac{1}{2}$, $66\frac{2}{3}$, $19\frac{3}{5}$.

(5 Reduce: 5 to 9ths; 16 to 3ds; 25 to 4ths; 24 to 10ths.

Miscellaneous

36 Oral:

(1) What will 8 yards of cloth cost at $1.50 a yard?

(2) At $6 a ton, how many tons of coal can be bought for $54?

(3) A square 10 inches on a side contains how many square inches?

(4) At 9 cents a quart, what will 3 gallons of milk sell for?

(5) If $\frac{1}{4}$ of a pound of cheese costs 10 cents, what will 2 pounds cost?

(6) Read DCXVII.

(7) If I have $25, and spend $\frac{2}{5}$ of it, how much do I spend?
(8) What must be added to 37 to make 50?
(9) How many days in 5 weeks?
(10) How many times is 6 contained in 366?

Written:

(1) Add 4086, 5946, 9468, 3875, 4998, and 5786.
(2) Divide the product of 77 and 72 by the product of 11 and 12.
(3) Change to lowest common denominator:

$$\frac{2}{15}, \frac{3}{10}, \frac{5}{6}, \frac{4}{5}.$$

(4) How much is received for 296 bushels of wheat at $0.95 a bushel?
(5) Write in Roman numerals: 119, 135, 142.
(6) What are the prime factors of 125?
(7) $\frac{2625}{25}$ = what?
(8) Change $35\frac{5}{6}$ to a fraction.
(9) What is 25% of 856?
(10) Add all the odd numbers between 10 and 30.

37 Oral:

(1) What is 50% of $100?
(2) Change $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$ to 18ths.
(3) $5\frac{2}{3}$ = how many thirds?
(4) Say the prime numbers from 1 to 20.

(5) What factor is common to 20, 30, and 50?
(6) At 50 cents an hour, what will a man earn in 6 eight-hour days?
(7) How many dozen eggs at 25 cents a dozen can be bought for $1.00?
(8) Change $\frac{96}{12}$ to a whole number.
(9) At 3 cents an ounce, what will a pound of pepper sell for?
(10) What coins could you get in exchange for a dollar?

38 Written:

(1) Subtract 97653 from 125000. Check result.
(2) If a railroad train goes 180 miles in 4 hours, how far could it go in 9 hours?
(3) The divisor is 349 and the quotient is 276. What is the dividend?
(4) Change to lowest terms: $\frac{24}{72}$, $\frac{50}{70}$, $\frac{32}{80}$.
(5) Find the highest common factor of 231 and 385.
(6) 1584 ounces = how many pounds?
(7) A lot is 45 feet long and 13 feet wide. What is its perimeter?
(8) How many steps of 2 feet each would a man take in walking a mile?
(9) Add $3.50, $152.63, $16.85, $125.18, and $9.75.
(10) How many square feet in a floor 65 feet long and 58 feet wide?

LESSON VI

Addition of Fractions

39 Add $\frac{3}{6}$ and $\frac{4}{6}$.

What is the denominator of the first fraction? The numerator? What does the 6 mean? The 3? The 4 in the second fraction?

Three parts, called sixths, are taken; then four parts, also called sixths, and of the same size, are taken. How many of this sized parts are taken in both fractions?

$\frac{3+4}{6}$ or $\frac{7}{6}$, which equal $1\frac{1}{6}$.

I. Add: $\frac{3}{8}$, $\frac{7}{8}$, $\frac{1}{8}$, $\frac{5}{8}$.

$$\frac{3+7+1+5}{8} = \frac{16}{8} = 2 = \text{sum.}$$

40 Oral exercise:

(1) $\frac{2}{4}+\frac{3}{4}+\frac{1}{4}+\frac{3}{4}=$what?
(2) Find the sum of $\frac{4}{9}$, $\frac{5}{9}$, $\frac{7}{9}$, $\frac{1}{9}$, $\frac{5}{9}$.
(3) Find the sum of $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{8}$, $\frac{7}{8}$.
(4) Find the sum of $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$.
(5) Add $\frac{11}{16}$, $\frac{9}{16}$, $\frac{4}{16}$, $\frac{5}{16}$.
(6) Add $\frac{3}{7}$, $\frac{7}{7}$, $\frac{6}{7}$, $\frac{5}{7}$.
(7) Add $\frac{2}{3}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{2}{3}$, $\frac{2}{3}$.
(8) How much are \$$\frac{2}{5}$, \$$\frac{4}{5}$, \$$\frac{3}{5}$, and \$$\frac{1}{5}$?
(9) To \$3 add \$$\frac{1}{5}$, \$$\frac{4}{5}$, \$$\frac{3}{5}$, \$$\frac{2}{5}$.

(10) Add 1 yard, $\frac{7}{8}$ yard, $\frac{3}{8}$ yard, and $\frac{5}{8}$ yard.
(11) Add 7 quarts, 9 quarts, $\frac{7}{8}$ quart and $\frac{6}{8}$ quart.
(12) Add $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{9}{7}$, $\frac{4}{7}$.

41 II. Add $\frac{1}{4}$ and $\frac{1}{5}$.

Here we are to add parts which are not of size, so we can not add the numerators. They will not equal 2 of any thing, any more than 1 pint and 1 quart will make 2 of any thing.

We must make of the $\frac{1}{4}$ and $\frac{1}{5}$ equivalent fractions having the same denominator.

$4 \times 5 = 20$
$20 = \text{L. C. D.}$

$$\left.\begin{array}{l} \dfrac{1 \times 5}{4 \times 5} = \frac{5}{20} \\[2ex] \dfrac{1 \times 4}{5 \times 4} = \frac{4}{20} \end{array}\right| = \frac{5+4}{20} = \frac{9}{20} = \text{sum.}$$

III. Add $\frac{1}{3}$, $\frac{5}{6}$, $\frac{7}{12}$.

Change to equivalent fractions having the same denominator.

$$\begin{array}{r|lll} 2 & 3 & 6 & 12 \\ \hline 3 & 3 & 3 & 6 \\ \hline & 1 & 1 & 2 \end{array}$$

$2 \times 3 \times 2 = 12$
$12 = \text{L. C. D.}$

$$\left.\begin{array}{l} \dfrac{1 \times 4}{3 \times 4} = \frac{4}{12} \\[2ex] \dfrac{5 \times 2}{6 \times 2} = \frac{10}{12} \\[2ex] \dfrac{7 \times 1}{12 \times 1} = \frac{7}{12} \end{array}\right| = \frac{4+10+7}{12} = \frac{21}{12}$$

$$\frac{21}{12} = 1\frac{9}{12} = 1\frac{3}{4} = \text{sum.}$$

Written exercise:

(1) Add $\frac{1}{2}$, $\frac{1}{4}$, $\frac{5}{8}$.

(2) Add $\frac{5}{6}$, $\frac{2}{3}$, $\frac{7}{12}$, $\frac{1}{2}$.

(3) Add \$$\frac{3}{4}$, \$$\frac{1}{2}$, \$$\frac{7}{8}$, \$$\frac{5}{12}$.

(4) I had $\frac{7}{12}$ yard of ribbon, and bought $\frac{5}{8}$ yard more. How many yards had I then?

(5) How many inches had I? How many more inches did I need to have 2 yards?

(6) Find the sum of $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{2}$, $\frac{11}{16}$.

(7) A boy walked $\frac{5}{8}$ mile the first hour, $\frac{11}{12}$ mile the second hour, $\frac{3}{4}$ mile the third hour, and $\frac{1}{2}$ mile the fourth hour. How far did he walk in the four hours?

(8) Mary had $\frac{7}{12}$ yard of ribbon; Anna had $\frac{5}{8}$ yard; Jessie had $\frac{2}{3}$ yard. How many yards had the three girls?

(9) $\frac{3}{4}+\frac{2}{3}+\frac{5}{6}+\frac{7}{8}=$ what sum?

(10) $\frac{1}{2}+\frac{4}{5}+\frac{9}{10}+\frac{3}{4}+\frac{1}{5}=$ what sum?

42 IV. Add: $2\frac{1}{2}$, $8\frac{3}{4}$, $9\frac{7}{8}$.

$$2\tfrac{1}{2}+8\tfrac{3}{4}+9\tfrac{7}{8}=2+8+9+2\tfrac{1}{8}=21\tfrac{1}{8}=\text{sum.}$$

$$\begin{array}{r|lll} 2 & 2 & 4 & 8 \\ \hline 2 & 1 & 2 & 4 \\ \hline & 1 & 1 & 2 \end{array}$$

$2\times2\times2=8$

$8=$ L. C. D.

$$\left.\begin{array}{l} \dfrac{1\times4}{2\times4}=\frac{4}{8} \\[2ex] \dfrac{3\times2}{4\times2}=\frac{6}{8} \\[2ex] \dfrac{7\times1}{8\times1}=\frac{7}{8} \end{array}\right| =\frac{4+6+7}{8}=\frac{17}{8}=2\tfrac{1}{8}$$

Rule: To add a number of fractions, or fractions and mixed numbers, change the fractional parts to equivalent fractions having the lowest common denominator; add the numerators, and set over the common denominator; change this fraction to its simplest form; then add this result to the sum of the integral parts, if there be any.

Written exercise:

(1) Add $2\frac{1}{3}$, $7\frac{5}{6}$, $9\frac{1}{2}$.
(2) Add $9\frac{4}{5}$, $2\frac{9}{10}$, $7\frac{11}{20}$.
(3) Add $9\frac{1}{8}$, $10\frac{5}{12}$, $7\frac{2}{3}$, $1\frac{3}{4}$.
(4) Find the sum of $\$2\frac{1}{2}$, $\$9\frac{3}{4}$, $\$7\frac{7}{8}$, $\$5\frac{11}{16}$.
(5) A boy rode on his bicycle $4\frac{1}{2}$ miles on Monday, $7\frac{3}{8}$ miles on Tuesday, $5\frac{7}{16}$ miles on Wednesday. How far did he ride on those three days?
(6) On the other three days of the week he rode $13\frac{1}{4}$ miles. How far was that in the whole week? (Sunday is excepted.)
(7) To $7\frac{7}{12}$ yards add $4\frac{1}{4}$ yards. To that sum add $9\frac{7}{8}$ yards.
(8) $9\frac{1}{6} + 4\frac{3}{4} + 7\frac{5}{16}$ = what sum?
(9) To the last answer add $2\frac{1}{2}$ and $1\frac{1}{3}$.
(10) In the barn I found $1\frac{1}{2}$ dozen eggs on Monday, $\frac{5}{6}$ dozen on Tuesday, $\frac{3}{4}$ dozen on Wednesday, $2\frac{2}{3}$ dozen in the rest of the week. How many dozen did I find in the whole week? How many eggs?

(1)	(2)	(3)	(4)	(5)
$+4\frac{1}{2}$	$+2\frac{7}{8}$	$+3\frac{1}{7}$	$+7\frac{1}{3}$	$+7\frac{4}{5}$
$3\frac{4}{5}$	$1\frac{11}{24}$	$2\frac{11}{14}$	$9\frac{4}{5}$	$2\frac{1}{2}$
$7\frac{9}{10}$	$3\frac{5}{6}$	$7\frac{1}{2}$	$7\frac{7}{15}$	$7\frac{9}{10}$
$3\frac{5}{6}$	$4\frac{5}{8}$	4	$3\frac{2}{3}$	$2\frac{3}{4}$

(6)	(7)	(8)	(9)	(10)
$+\frac{1}{6}$	$+\frac{7}{8}$	$+\frac{11}{12}$	$+\frac{19}{20}$	$+\frac{8}{9}$
$\frac{4}{5}$	$\frac{3}{4}$	$\frac{23}{24}$	$\frac{4}{5}$	$\frac{2}{3}$
$\frac{14}{15}$	$\frac{15}{16}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{9}$
$\frac{9}{10}$	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{9}{10}$	$\frac{23}{27}$

43 Oral—Add:

(1)	(2)	(3)	(4)	(5)	(6)
$2\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$9\frac{1}{3}$	$1\frac{1}{2}$	$2\frac{7}{8}$
$3\frac{1}{3}$	$3\frac{3}{4}$	$3\frac{1}{5}$	$10\frac{2}{3}$	$3\frac{5}{6}$	$7\frac{3}{4}$
$7\frac{5}{6}$	$1\frac{7}{8}$	$7\frac{4}{5}$	$7\frac{8}{9}$	$2\frac{1}{3}$	$2\frac{1}{2}$

(7)	(8)	(9)	(10)	(11)	(12)
$5\frac{4}{5}$	$1\frac{2}{3}$	$2\frac{1}{2}$	$1\frac{3}{4}$	$7\frac{1}{2}$	$1\frac{1}{2}$
$2\frac{1}{2}$	$1\frac{3}{4}$	$3\frac{5}{6}$	$2\frac{11}{12}$	$3\frac{1}{3}$	$2\frac{2}{3}$
$7\frac{3}{10}$	$1\frac{5}{6}$	$7\frac{2}{3}$	$3\frac{1}{2}$	$10\frac{1}{4}$	$7\frac{5}{12}$

44 Written:

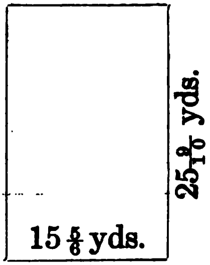
1. $1\frac{1}{2}$, $3\frac{1}{3}$, $9\frac{1}{5}$, $2\frac{11}{15}$, $1\frac{2}{3}$ Note: Add horizontally.
2. $2\frac{5}{6}$, $3\frac{1}{8}$, $10\frac{2}{3}$, $8\frac{1}{2}$, $3\frac{5}{8}$ Add vertically.
3. $9\frac{3}{4}$, $3\frac{7}{24}$, $2\frac{1}{2}$, $2\frac{2}{3}$, $2\frac{5}{6}$
4. $7\frac{5}{8}$, $2\frac{2}{3}$, $7\frac{5}{6}$, $3\frac{3}{4}$, $11\frac{11}{24}$
5. $3\frac{1}{2}$, $7\frac{5}{6}$, $2\frac{7}{10}$, $9\frac{7}{30}$, $1\frac{2}{3}$

45 (1) Add $\frac{1}{2}$ and $\frac{7}{8}$.

(2) To the sum add $\frac{9}{16}$. Then to that sum add $\frac{5}{24}$.

(3) A man had \$100$\frac{1}{5}$; he earned \$216$\frac{7}{8}$ and \$40$\frac{4}{5}$. How much had he then?

(4) A square is $9\frac{5}{16}$ yards on each side. How far is it around it?



(5) How far is it around this oblong?

(6) A boy walked around it twice. How far was that?

(7) If it were twice as long, what would its perimeter be?

(8) If it were three times as wide, what would its perimeter be?

(9) Make a square $2\frac{3}{4}$ inches on a side. What is its perimeter?

(10) Make one three times as long on a side. What is its perimeter?

LESSON VII

Subtraction of Fractions

46 In subtraction we must remember TWO things: The first, that the minuend must be larger than the subtrahend; and the second, that these two terms must be of the same denomination. For instance, 7 cents from 10 cents, 7 quarts from 10 quarts, 7 twelfths from 10 twelfths. This is as true of fractional as of integral quantities.

1. From $\frac{10}{12}$ take $\frac{7}{12}$ $\frac{10-7}{12} = \frac{3}{12} = \frac{1}{4}$ = the remainder.

This means that from 10 parts, called twelfths, you must subtract 7 of the same sized parts.

2. From $\frac{5}{6}$ take $\frac{3}{4}$.

Here the fractions must first be made into equivalent fractions with the same, or a common, denominator before you subtract the numerators; just as you could not take 3 pints from 5 quarts and leave 2 of any thing; you would first change to the same denomination, and then subtract.

$$\frac{5}{6} - \frac{3}{4}.$$

$$\begin{array}{l} 2\,\underline{)6 - 4} \\ \quad 3 - 2 \end{array} \qquad \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

$$= \frac{10-9}{12} = \frac{1}{12} = \text{the remainder.}$$

$$2 \times 3 \times 2 = 12$$

$$12 = \text{L. C. D.} \qquad \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

47 Oral exercise:

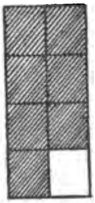
Note.—Always give the answer in the simplest form.

Take $\frac{1}{7}$ from $\frac{6}{7}$; $\frac{2}{3}-\frac{1}{2}=?$ $\frac{1}{2}-\frac{1}{6}=?$ $\frac{2}{3}-\frac{1}{4}=?$
$\frac{3}{10}$ from $\frac{8}{10}$; $\frac{10}{12}-\frac{1}{2}=?$ $\frac{1}{2}-\frac{1}{4}=?$ $\frac{2}{3}-\frac{1}{6}=?$
$\frac{4}{11}$ from $\frac{10}{11}$; $\frac{15}{16}-\frac{1}{2}=?$ $\frac{1}{2}-\frac{1}{3}=?$ $\frac{2}{3}-\frac{1}{12}=?$
$\frac{3}{12}$ from $\frac{9}{12}$; $\frac{9}{10}-\frac{1}{2}=?$ $\frac{1}{2}-\frac{1}{5}=?$ $\frac{2}{3}-\frac{1}{5}=?$
$\frac{1}{6}$ from $\frac{5}{6}$; $\frac{7}{12}-\frac{1}{2}=?$ $\frac{1}{2}-\frac{1}{7}=?$ $\frac{2}{3}-\frac{1}{9}=?$

48 Written exercise:

1. $\frac{5}{12}-\frac{1}{3}=?$
2. $\frac{7}{12}-\frac{1}{2}=?$
3. $\frac{9}{14}-\frac{2}{7}=?$
4. $\frac{25}{27}-\frac{2}{3}=?$
5. $\frac{50}{51}-\frac{2}{3}=?$
6. $\frac{8}{9}-\frac{1}{27}=?$
7. $\frac{9}{15}-\frac{7}{30}=?$
8. $\frac{14}{17}-\frac{3}{51}=?$
9. $\frac{19}{20}-\frac{2}{3}=?$
10. $\frac{14}{15}-\frac{2}{3}=?$
11. Take $\frac{1}{6}$ from $\frac{19}{30}$
$\frac{3}{5}$ $\frac{19}{20}$
$\frac{7}{12}$ $\frac{14}{15}$
$\frac{9}{15}$ $\frac{2}{3}$
$\frac{1}{4}$ $\frac{13}{15}$
$\frac{5}{12}$ $\frac{49}{60}$

49 1 (or $\frac{8}{8}$) $-\frac{7}{8}$ (1) From 1 take $\frac{7}{8}$.



Here again you must make the terms of the same denomination before you subtract.

$$1=\frac{8}{8} \quad -\frac{7}{8} \;\Big|\; =\frac{8-7}{8}=\frac{1}{8}=\text{the remainder.}$$

(2) From 13 take $1\frac{7}{8}$.

Here take 1 unit from the 13 units, and make a fraction of the same denomination as the subtrahend.

$$13=12\frac{8}{8} \quad -1\frac{7}{8} \;\Big|\; =11\frac{8-7}{8}=11\frac{1}{8}=\text{remainder.}$$

50 Oral:

From 1 take, in turn, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{9}{12}$, $\frac{4}{11}$, $\frac{6}{13}$, $\frac{9}{10}$, $\frac{1}{4}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{19}{20}$.
From 12 take, in turn, the same subtrahends.

Written exercise:

1. $2-1\frac{2}{3}$
2. $7-6\frac{1}{4}$
3. $10-7\frac{5}{6}$ =?
4. $12-4\frac{5}{17}$
5. $10-3\frac{4}{15}$
6. $10-6\frac{1}{4}$
7. $100-16\frac{2}{3}$
8. $100-94\frac{1}{7}$
9. $100-63\frac{2}{15}$ =?
10. $100-14\frac{19}{20}$
11. $100-60\frac{4}{9}$
12. $100-20\frac{19}{30}$
13. $4\frac{9}{20}$
14. $17\frac{3}{25}$
15. $50-19\frac{14}{30}$ =?
16. $43\frac{16}{50}$
17. $49\frac{19}{30}$
18. $7\frac{1}{40}$

(1) How much will be left from \$5, if you spend \$$3\frac{9}{10}$?

(2) A boy had to travel 50 miles. How many more miles did he have to go after he had gone $20\frac{9}{20}$ miles?

(3) How much more is $\frac{5}{7}$ than $\frac{5}{21}$?

(4) How much less is $\frac{1}{10}$ than $\frac{4}{7}$?

(5) Take $\frac{7}{15}$ from $\frac{7}{9}$. Take $2\frac{7}{15}$ from 19. Add the two remainders.

(6) Who has more ribbon, Mary, who has $\frac{3}{4}$ of a yard, or Emma, who has $\frac{7}{12}$ of a yard? How much more?

(7) Add $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. Then subtract $\frac{3}{8}$ from the sum.

(8) $(\frac{1}{6}+\frac{1}{8}+\frac{1}{4})-\frac{5}{12}$ =what remainder?

(9) $(\frac{1}{2}+\frac{1}{5})-(\frac{1}{3}+\frac{1}{10})$ =what remainder?

(10) Subtract $\frac{2}{5}$ from $\frac{9}{10}$; subtract $\frac{1}{2}$ from $\frac{2}{3}$; then add the two remainders.

To take a fraction or a mixed number from a mixed number:

From $7\frac{1}{2}$ take $2\frac{1}{5}$.

Here you see at once that the fractional parts are not of the same denomination. You must first change them to equivalent fractions of a common denominator.

$2\times5=10$
$10=$ L.C.D.

$$\frac{1\times5}{2\times5}=\frac{5}{10} \quad 7\frac{5}{10}$$

$$\frac{1\times2}{5\times2}=\frac{2}{10} \quad 2\frac{2}{10}$$

$=5\frac{5-2}{10}=5\frac{3}{10}=$ the remainder.

51 Oral exercise:

$6\frac{1}{2}-2\frac{1}{3}$
$12\frac{4}{5}-3\frac{1}{5}$
$7\frac{3}{8}-1\frac{1}{4}$
$6\frac{11}{12}-1\frac{1}{2}$
$7\frac{7}{8}-2\frac{1}{2}$

$13\frac{1}{2}-7\frac{2}{5}$
$6\frac{1}{3}-2\frac{1}{6}$
$12\frac{1}{4}-6\frac{1}{8}$
$9\frac{2}{5}-1\frac{3}{10}$
$6\frac{1}{4}-1\frac{1}{5}$

$2\frac{4}{9}-1\frac{1}{3}$
$17\frac{3}{5}-2\frac{1}{10}$
$16\frac{1}{2}-7\frac{3}{7}$
$9\frac{5}{6}-2\frac{4}{9}$
$1\frac{4}{5}-\frac{7}{10}$

52 Written exercise:

From $119\frac{5}{6}$ take $14\frac{3}{7}$
$93\frac{7}{15}$
$25\frac{3}{16}$
$43\frac{11}{16}$
$20\frac{50}{60}$

$(7\frac{4}{9}-2\frac{5}{18})+7\frac{4}{5}$
$(2\frac{7}{11}-1\frac{9}{22})+3\frac{1}{3}$
$(7\frac{4}{5}-2\frac{17}{25})+6\frac{1}{2}$
$(17\frac{4}{12}-2\frac{1}{3})+3\frac{11}{18}$
$(7\frac{9}{10}-2\frac{39}{50})+7\frac{17}{25}$

(1) A lady had $7\frac{1}{2}$ yards of lace; she used $2\frac{1}{3}$ yards, then bought $5\frac{3}{4}$ yards more. How many had she then?

(2) A boy had $\$9\frac{9}{10}$; he spent \$3 one day, and $\$4\frac{7}{20}$ another day. What had he left?

(3) A barrel holds 196 pounds of flour. If $117\frac{1}{2}$ pounds are used, and then $16\frac{2}{3}$ pounds are used, how many pounds remain?

(4) Add $\$6\frac{1}{2}$, $\$7\frac{3}{4}$, $\$9\frac{7}{8}$, and take the sum from $\$25\frac{9}{16}$.

(5) John and Henry are saving to buy bicycles just alike. Who has more and how much, John with $\frac{9}{14}$ of the required amount, or Henry with $\frac{11}{21}$?

53 From $17\frac{1}{3}$ take $4\frac{5}{6}$.

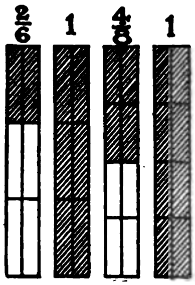
What is the first thing to do here? (L.C.D.)

$$\begin{array}{l} 3\underline{)3-6} \\ \quad 1-2 \\ 3\times2=6 \\ \quad 6=\text{L.C.D.} \end{array} \qquad \frac{1\times2}{3\times2}=\frac{2}{6} \qquad \frac{5\times1}{6\times1}=\frac{5}{6}$$

Then you have $17\frac{2}{6}-4\frac{5}{6}$.

What is the second thing you must always remember in subtraction? (Minuend larger than subtrahend.)

In this case you see you must "carry" one, just as you do with integers when you need to. You must take 1 unit from the 17 units, and change it to sixths. You will then have $16\frac{8}{6}-4\frac{5}{6}$.



$$\left.\begin{array}{r} 17\frac{2}{6}=16\frac{8}{6} \\ -4\frac{5}{6}-\ \ 4\frac{5}{6} \end{array}\right| =12\frac{8-5}{6}=12\frac{3}{6}=12\frac{1}{2}=\text{the remainder.}$$

Try another. From $6\frac{1}{2}$ take $2\frac{7}{8}$.

$$\begin{array}{ll|l}
2)\underline{2\text{-}8} & \frac{1}{2}\times\frac{4}{4}=\frac{4}{8},\quad 6\frac{4}{8}=5\frac{12}{8} & \\
\;\;\;1\text{-}4 & & \\
 & & =3\frac{12-7}{8}=3\frac{5}{8}= \\
2\times4=8 & & \text{the remainder.} \\
\;\;\;\;\;\;8=\text{L.C.D.} & \frac{7}{8}\times\frac{1}{1}=\frac{7}{8},\quad 2\frac{7}{3}=2\frac{7}{8} &
\end{array}$$

54 Written exercise:

$$\left.\begin{array}{r}
3\frac{1}{2}-2\frac{4}{5} \\
7\frac{1}{2}-3\frac{2}{3} \\
2\frac{1}{4}-1\frac{5}{8} \\
10\frac{1}{5}-6\frac{9}{10} \\
8\frac{1}{3}-3\frac{7}{12}
\end{array}\right| =? \qquad\qquad
\left.\begin{array}{r}
9\frac{5}{12}-\;\;6\frac{7}{8} \\
2\frac{1}{10}-\;\;1\frac{4}{15} \\
11\frac{3}{17}-\;\;4\frac{1}{2} \\
17\frac{3}{5}-\;\;8\frac{7}{8} \\
16\frac{3}{11}-10\frac{3}{4}
\end{array}\right| =?$$

$$76\frac{1}{3}-\left|\begin{array}{l}
25\frac{4}{15} \\
16\frac{9}{13} \\
53\frac{17}{43} \\
29\frac{19}{20} \\
68\frac{10}{11}
\end{array}\right. =? \qquad\qquad
\left.\begin{array}{l}
19\frac{1}{7} \\
16\frac{1}{3} \\
93\frac{11}{50} \\
14\frac{3}{10} \\
25\frac{17}{36}
\end{array}\right| -18\frac{1}{2}=$$

(1) A boy had $50\frac{1}{3}$ yards of kite string. His kite broke away and carried off $29\frac{7}{8}$ yards. How much had he left?

(2) How much did he need to get to make just 60 yards?

(3) A lady used $7\frac{5}{16}$ yards of lace for one garment, and $9\frac{7}{8}$ yards for another. How much had she left from $20\frac{1}{3}$ yards?

(4) It is $98\frac{1}{5}$ miles from Baltimore to Philadelphia. John traveled on his wheel $16\frac{7}{20}$ miles on Monday, $20\frac{4}{5}$ miles on Tuesday, $30\frac{7}{8}$ miles on Wednesday, and the rest on Thursday. How far did he ride on Thursday?

(5) $(\$9\frac{1}{2} + \$16\frac{3}{4} + \$24\frac{1}{3} + \$17\frac{11}{12}) - \$29\frac{23}{24}$ = what remainder?

(6) Take the sum of $4\frac{5}{27}$, $3\frac{8}{9}$, $2\frac{2}{3}$ from $95\frac{1}{2}$.

(7) How much less is $20\frac{11}{16}$ than $40\frac{1}{3}$?

(8) How much more is $19\frac{1}{7}$ than $9\frac{11}{42}$?

(9) The minuend is $100\frac{1}{6}$, the subtrahend is $89\frac{17}{18}$. What is the remainder?

(10) The minuend is $117\frac{4}{25}$, the remainder is $69\frac{49}{50}$. What is the subtrahend?

LESSON VIII

Notation and Numeration

55 Definitions:

Notation is the writing of numbers by means of symbols.

Numeration is the reading of numbers which are written in symbols.

There are two systems of notation: Roman and Arabic.

In the Roman system the symbols used are the capital letters, I, V, X, L, C, D, M. What value does each represent?

In order to read Roman numerals we must know certain rules:

(1) Repeating a letter repeats its value. XXX = 30.

(2) Writing a smaller value *after* a larger adds it to the larger. XV = 15.

(3) Writing a smaller value *before* a larger subtracts it from the larger. XL = 40.

Read: MDC, CCC, LXXV, XLIX, CD, XLV, XCII.

Write the numbers from 1 to 10; 20; 30; 40 to 100.

In the Arabic system the symbols are the figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Of these 0 expresses no value, but is used in writing numbers of more than one figure. It is called *Naught*, *Zero*, or a *Cipher*. Each of the other figures expresses a value, which depends upon its place in the number. Thus:

5 = five.
50 = fifty (five tens).
500 = five hundred.
5000 = five thousand.

By combining these figures from 1 to 9, called *digits*, and the cipher we can write very large numbers.

hundred billions	ten billions	billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	units
											1
										1	0
									1	0	0
								1	0	0	0
							1	0	0	0	0
						1	0	0	0	0	0
					1	0	0	0	0	0	0
				1	0	0	0	0	0	0	0
			1	0	0	0	0	0	0	0	0
		1	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0
4th or billions period			3d or millions period			2d or thousands period			1st or units period		

Read each number written above.

Read the following:

605
49
370
850
28
700

3,090
10,980
43,027
8,506
75,680
923,046

500,000,000
62,850,075
1,050,808
25,400,400
670,584,720
90,500,000

2,500,500,500
12,800,800,800
5,005,005,005
60,750,750,750
125,125,125,125
68,735,406,780

56 Write the following in figures:

(1) Four hundred thirty-six.
(2) Twenty-four thousand, twenty-four.
(3) Eighty-three million.
(4) One thousand, nine hundred fifteen.
(5) Three hundred seventy-five billion.
(6) Seven hundred million, seven hundred thousand, seven hundred.
(7) One million, five hundred thousand.
(8) Five hundred ten thousand, four hundred five.
(9) A number containing 2 in the millions place, 2 in the thousands place, 2 in the units place, ciphers in the vacant places. Read it.

A unit is a single thing. What is the largest number of units that can be expressed by a single figure? How many units make one ten? Then 1 of the tens order is ten times as great as 1 of the units order. How many tens make one hundred? Then 1 of the hundreds order is ten times as great as 1 of the tens order. How many hundreds make one thousand? Then 1 of the thousands order is how many times as great as 1 of the hundreds order?

Since	1 of the tens order	is 10 times as great as	1 of the units order
	1 of the hundreds order		1 of the tens order
	1 of the thousands order		1 of the hundreds order,

Then

Moving a figure	one place to the left	multiplies its value by	10
	two places to the left		100
	three places to the left		1000

Rule:

To multiply a number by 10, annex one cipher.
To multiply a number by 100, annex two ciphers.
To multiply a number by 1000, annex three ciphers.

57 Multiply 85 by 10; by 100; by 1000.
Multiply 276 by 10; by 100; by 1000.

25	×10;	16	×100;	87	×1000;	36	×20
36		29		60		50	
47		30		54		14	
53		50		12		72	

Moving a figure	one place to the right	divides its value by	10
	two places to the right		100
	three places to the right		1000

Rule:

To divide a number by 10, cut off the units figure	from
To divide a number by 100, cut off the units and tens figures	the
To divide a number by 1000, cut off the units, tens, and hundreds figures	dividend.

10)58|5
58 = Quo., 5 = Rem.

100)6|75
6 = Quo., 75 = Rem.

1000)7|685
7 = Quo., 685 = Rem.

Divide 85 by 10; 685 by 100; 8685 by 1000.

956	÷10;	593	÷100;	3864	÷1000;	3500	÷	10
374		678		2000		6750		100
500		1574		9600		8375		1000
275		2697		4735		9150		

LESSON IX

FOURTH GRADE **SECOND HALF YEAR**

Review of Multiplication, Division, Factors, Fractions, and Definitions.

58 I. What is multiplication? What are the terms used in multiplication? Define each one. Show how multiplication is a short way to do addition.

Find 8 times 6985, in two ways.

How can you check problems in multiplication?

Oral work:

1. 7 × 15
90
81
75
63
18 = ?
14
95
16
64

2. 8 × 90
91
75
63
18
67 = ?
49
73
70
54

3. 9 × 40
50
500
64
600
75 = ?
700
16
160
86

4. 12 × 30
33
20
200
50
51 = ?
500
60
700
44

Written exercise:

5.
$$\left.\begin{array}{r}5\\6\\7\\8\\9\\10\\11\\12\end{array}\right| \times \left|\begin{array}{r}12\\0\\11\\10\\2\\9\\3\\8\\4\\7\\5\\6\end{array}\right. = ?$$

6. 983 times 12307
7. 745 4984
8. 806 3617
9. 475 8906
10. 689 5427 Check each.
11. 237 2139
12. 849 1807
13. 298 7654
14. 150 1239
15. 647 2078

59 II. What is division? What terms are used in division? Define each one. How can you check problems in division? Show three ways of expressing division. ($5\overline{)40}$; $40 \div 5$; $\frac{40}{5}$).

(Dividing a number by 5 is the same as taking $\frac{1}{5}$ of the number; so *dividing* by any *number* is *multiplying* by the *number inverted*.)

Oral exercise:

$$\left.\begin{array}{rr} 1. & 693 \\ 2. & 478 \\ 3. & 530 \\ 4. & 705 \\ 5. & 689 \\ 6. & 438 \\ 7. & 873 \\ 8. & 498 \\ 9. & 764 \\ 10. & 207 \end{array}\right| \div \left|\begin{array}{l} 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array}\right. = ?$$

$$11.\quad 1000 \div \left|\begin{array}{r} 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{array}\right. = ?$$

$$12.\quad \text{What is } \left.\begin{array}{c} \frac{1}{2} \\ \frac{2}{3} \\ \frac{4}{5} \\ \frac{7}{8} \\ \frac{5}{6} \\ \frac{5}{8} \\ \frac{3}{4} \\ \frac{9}{10} \end{array}\right| \text{ of } \left.\begin{array}{r} 600 \\ 1200 \\ 480 \\ 240 \\ 720 \end{array}\right| ?$$

60 Written exercise:

$$\left.\begin{array}{rr} 1. & 1000001 \\ 2. & 9867543 \\ 3. & 2170049 \\ 4. & 1002005 \\ 5. & 9813469 \\ 6. & 4123500 \\ 7. & 4190000 \\ 8. & 6170123 \\ 9. & 7168914 \\ 10. & 2001456 \end{array}\right| \div \left|\begin{array}{l} 416 \\ 250 \\ 783 \\ 425 \\ 125 \\ 276 \\ 987 \\ 409 \\ 607 \\ 819 \end{array}\right. = ?$$

Check each problem. KNOW that it is correctly solved.

61 III. What is a product? What are factors?

Factor × factor = product.

Product ÷ factor = other factor.

Name factors of:

60, 42, 9, 25, 36, 54, 64, 72, 80, 100, 16, 18, 24, 32, 45, 63, 12, 49, 99, 84, 75, 105.

If the product is	and one factor is	what is the other factor?
63	7	
24	8	
27	3	
54	6	
75	5	
45	3	
105	15	
90	5	
40	8	
72	9	

What are prime numbers? Name the prime numbers up to 50. What are prime factors? Find the prime factors of the products given in the section above.

What are common factors? Find the common prime factors of:

63, 45; 72, 96; 100, 150; 80, 45; 27, 54; 16, 40;
50, 60; 84, 24; 70, 42; 49, 91; 36, 72; 18, 90.

What is the highest common factor? How is it found?

Find the H. C. F. of:

12, 16; 25, 35; 24, 48; 56, 16; 46, 69;
49, 35; 30, 40; 36, 48; 28, 70; 34, 51.

What is a multiple? What is a common multiple?

What is the lowest common multiple? How is it found?

Find the L. C. M. of:

12, 16, 24; 8, 16, 12; 5, 10, 25; 4, 9, 16;
7, 14, 21; 6, 12, 18; 5, 10, 15; 8, 16, 24;
9, 18, 27; 10, 20, 30; 8, 5, 16; 3, 4, 5;
2, 9, 12; 3, 9, 12; 4, 9, 12; 5, 9, 12.

62 IV. What is a fraction? What are the terms of a fraction?

Explain what each term means. How do the terms correspond to the terms of division?

What are equivalent fractions? Show by diagrams how $\frac{1}{2}$ can be changed to 10ths, 8ths, 12ths, 16ths.
$\frac{1}{3}$ to 12ths, 15ths, 9ths, 6ths.
$\frac{1}{5}$ to 10ths, 15ths.
$\frac{1}{4}$ to 8ths, 12ths, 16ths, 20ths.

What must be done with fractions before they can be added?

Add:	Find the sums of:
$6\frac{1}{2}$, $2\frac{1}{3}$, $4\frac{1}{9}$, $8\frac{1}{6}$	$9\frac{1}{9}$, $2\frac{7}{18}$, $3\frac{31}{36}$, and 7
$7\frac{2}{3}$, $8\frac{3}{4}$, $9\frac{4}{5}$, $8\frac{5}{6}$	$6\frac{4}{5}$, $7\frac{9}{10}$, $6\frac{4}{15}$, and 3
$2\frac{1}{7}$, $8\frac{1}{3}$, $9\frac{9}{14}$, $8\frac{11}{21}$	$7\frac{1}{2}$, 9 , $4\frac{5}{15}$, and $9\frac{7}{26}$
$6\frac{1}{5}$, $7\frac{7}{10}$, $6\frac{14}{15}$, $5\frac{5}{6}$	$4\frac{3}{7}$, $2\frac{4}{5}$, $17\frac{34}{35}$, and $16\frac{6}{7}$

63 V. What two things must you remember when subtracting one fractional quantity from another? What are the terms in subtraction? Explain what each means. How do you check subtraction?

64 Oral exercise:

From	1	subtract	$\frac{4}{5}$
	2		$\frac{9}{20}$
	3		$\frac{7}{12}$
	4		$\frac{6}{7}$
	5		$\frac{8}{15}$
	7		$\frac{12}{13}$
	9		$\frac{10}{13}$
	10		$\frac{9}{16}$
	12		$\frac{19}{25}$
	15		$\frac{7}{24}$

65 Written exercise:

$$\begin{array}{r} 9\frac{1}{2} \\ -\ 7\frac{3}{4} \\ \hline \end{array} \qquad \begin{array}{r} 7\frac{2}{3} \\ -\ 2\frac{4}{5} \\ \hline \end{array} \qquad \begin{array}{r} 6\frac{1}{3} \\ -\ 1\frac{9}{10} \\ \hline \end{array}$$

$$\begin{array}{r} 17\frac{4}{9} \\ -\ 2\frac{19}{27} \\ \hline \end{array} \qquad \begin{array}{r} 16\frac{5}{7} \\ -\ 4\frac{19}{35} \\ \hline \end{array} \qquad \begin{array}{r} 70\frac{3}{14} \\ -\ 7\frac{31}{35} \\ \hline \end{array}$$

$(9\frac{1}{2}+6\frac{2}{3})-(4\frac{1}{4}+7\frac{5}{12})=?$

$(9\frac{2}{3}+7)-(9-2\frac{7}{8})=?$

From $19\frac{1}{5}$ take $(9\frac{4}{7}+7\frac{1}{5})$.

From (7×63) take $(195\frac{1}{4}+70\frac{5}{6})$.

How much less is $97\frac{1}{2}$ than $105\frac{2}{9}$?

$68\frac{1}{5}$ than $143\frac{1}{11}$?

How much greater is 5×75 than $49\frac{7}{15}$?

16×47 than $129\frac{7}{60}$?

$(15\frac{1}{4}+26\frac{9}{10})$ than $30\frac{17}{40}$?

LESSON X

Multiplication of Fractions

66 2 times 6 sevenths? 2 times 6 of anything will make 12 of that thing. $(2 = \frac{2}{1}.)$

$$2 \times \frac{6}{7} = \frac{2}{1} \times \frac{6}{7} = \frac{12}{7} = 1\frac{5}{7} = \text{Product.}$$

This process is shortened by cancellation. Thus:

$$2 \times \frac{5}{6} = \frac{2}{1} \times \frac{5}{6} = \frac{5}{3} = 1\frac{2}{3} = \text{Product.}$$

Oral exercise:

$$\left.\begin{array}{c}2\\3\\4\\6\\7\\8\\9\end{array}\right| \text{times } \frac{4}{5}; \quad \left.\begin{array}{c}2\\3\\4\\5\\6\\8\\9\end{array}\right| \text{times } \frac{3}{7}; \quad 4 \text{ times} \left|\begin{array}{c}\frac{1}{2}\\\frac{3}{4}\\\frac{7}{8}\\\frac{5}{12}\end{array}\right. ; \quad 8 \text{ times} \left|\begin{array}{c}\frac{1}{2}\\\frac{3}{4}\\\frac{5}{8}\\\frac{7}{16}\end{array}\right.$$

$$\left.\begin{array}{l}2 \text{ times } \frac{6}{7}\\ \frac{6}{7} \text{ times } 2\\ \frac{6}{7} \text{ of } 2\end{array}\right\}$$

These are different ways of indicating the product of two factors, 2 and $\frac{6}{7}$.

$$\left.\begin{array}{l}\frac{2}{1} \times \frac{6}{7} = \frac{2 \times 6}{1 \times 7}\\ \frac{6}{7} \times \frac{2}{1} = \frac{6 \times 2}{7 \times 1}\\ \frac{6}{7} \text{ of } \frac{2}{1} = \frac{6 \times 2}{7 \times 1}\end{array}\right| = \frac{12}{7} = 1\frac{5}{7} = \text{Product.}$$

67 Oral exercise:

$$\frac{6}{7}\text{ times}\left|\begin{array}{l}3\\4\\5\\6\\8\\9\end{array}\right. ;\quad \frac{5}{6}\text{ times}\left|\begin{array}{r}6\\12\\18\\24\\30\end{array}\right. ;\quad \frac{7}{12}\text{ times}\left|\begin{array}{r}2\\3\\4\\6\\12\end{array}\right. ;\quad \frac{3}{4}\text{ times}\left|\begin{array}{r}20\\12\\8\\4\\16\end{array}\right.$$

68 Written exercise:

$$9\times\left|\begin{array}{c}\frac{4}{5}\\\frac{2}{7}\\\frac{3}{10}\\\frac{5}{8}\\\frac{3}{4}\end{array}\right. ;\ \frac{7}{9}\times\left|\begin{array}{r}5\\8\\7\\18\\27\\45\\36\end{array}\right. ;\quad 12\times\left|\begin{array}{c}\frac{1}{2}\\\frac{3}{4}\\\frac{5}{6}\\\frac{2}{3}\\\frac{4}{5}\end{array}\right. ;\ \left.\begin{array}{c}\frac{1}{2}\\\frac{3}{4}\\\frac{2}{5}\\\frac{7}{10}\\\frac{9}{20}\end{array}\right|\text{ of }\begin{array}{c}\$1\\\text{or}\\100\\\text{cts.}\end{array} ;\ \left.\begin{array}{c}\frac{4}{9}\\\frac{5}{12}\\\frac{3}{8}\\\frac{5}{6}\\\frac{3}{4}\end{array}\right|\begin{array}{c}\text{1 sq. ft.}\\\text{or}\\\text{of 144}\\\text{sq. in.}\end{array}$$

These are different ways of indicating the product of two factors, both being fractions.

$$\left.\begin{array}{ccc}\frac{6}{7}&\text{times}&\frac{3}{4}\\\frac{3}{4}&\text{times}&\frac{6}{7}\\\frac{6}{7}&\text{of}&\frac{3}{4}\\\frac{3}{4}&\text{of}&\frac{6}{7}\end{array}\right|\quad \frac{\overset{3}{\cancel{6}}}{7}\times\frac{3}{\underset{2}{\cancel{4}}}=\frac{3}{7}\times\frac{3}{2}=\frac{9}{14}=\text{Product.}$$

69 Oral exercise:

$$\left.\begin{array}{c}\frac{1}{4}\\\frac{1}{5}\\\frac{1}{6}\\\frac{1}{7}\\\frac{1}{8}\\\frac{1}{9}\end{array}\right|\text{ of }\left|\begin{array}{c}\frac{2}{3}\\\frac{1}{4}\\\frac{5}{9}\\\frac{3}{7}\\\frac{4}{5}\\\frac{7}{8}\end{array}\right. ;\quad \begin{array}{ccc}\frac{1}{2}&\times&\frac{2}{3}\\\frac{3}{4}&\times&\frac{8}{9}\\\frac{2}{5}&\text{of}&\frac{5}{14}\\\frac{5}{6}&\text{of}&\frac{18}{25}\\\frac{2}{3}&\text{of}&\frac{9}{10}\end{array} ;\quad \begin{array}{ccc}\frac{1}{6}&\times&\frac{6}{7}\\\frac{2}{7}&\times&\frac{14}{15}\\\frac{5}{6}&\times&\frac{12}{15}\\\frac{6}{7}&\times&\frac{1}{3}\\\frac{8}{9}&\text{of}&\frac{3}{4}\end{array} ;\quad \left.\begin{array}{c}\frac{1}{2}\\\frac{1}{3}\\\frac{1}{4}\\\frac{1}{5}\\\frac{1}{6}\end{array}\right|\text{ of }\left|\begin{array}{c}\frac{1}{6}\\\frac{1}{5}\\\frac{1}{4}\\\frac{1}{3}\\\frac{1}{2}\end{array}\right.$$

70 Written exercise:

$\frac{3}{5} \times \frac{8}{9}$
$\frac{6}{7} \times \frac{5}{11}$
$\frac{8}{13} \times \frac{2}{3}$
$\frac{9}{14} \times \frac{3}{4}$

$\frac{7}{10} \times \frac{15}{28}$
$\frac{9}{14} \times \frac{21}{36}$
$\frac{6}{11} \times \frac{33}{54}$
$\frac{5}{9} \times \frac{27}{50}$

$\frac{10}{21} \times \frac{14}{15}$
$\frac{7}{20} \times \frac{30}{49}$
$\frac{5}{8} \times \frac{24}{35}$
$\frac{7}{12} \times \frac{18}{49}$

$\frac{2}{3}$ of $\frac{6}{11}$ of $\frac{22}{27}$
$\frac{4}{5}$ of $\frac{20}{21}$ of $\frac{15}{16}$
$\frac{3}{7}$ of $\frac{1}{9}$ of $\frac{14}{15}$
$\frac{3}{8} \times \frac{4}{5}$ of $\frac{25}{36}$

$2\frac{6}{7} \times \frac{3}{4}$
$\frac{3}{4}$ of $2\frac{6}{7}$
$2\frac{6}{7} \times 3\frac{3}{4}$

These are different ways of indicating the product of two factors, both containing a fraction. Change the mixed numbers to fractions.

$$\frac{\overset{5}{\cancel{20}}}{7} \times \frac{3}{\cancel{4}} = \frac{5}{7} \times \frac{3}{1} = \frac{15}{7} = 2\frac{1}{2} = \text{Product.}$$

$$\frac{\overset{5}{\cancel{20}}}{7} \times \frac{15}{\cancel{4}} = \frac{5}{7} \times \frac{15}{1} = \frac{75}{7} = 10\frac{5}{7} = \text{Product.}$$

71 Oral exercise:

$$\left.\begin{array}{l} 1\frac{1}{3} \\ 2\frac{1}{4} \\ 3\frac{1}{2} \\ 1\frac{1}{2} \\ 2\frac{2}{3} \\ 4\frac{2}{7} \end{array}\right| \times \frac{1}{4} \quad ; \quad \frac{1}{5} \text{ of } \left|\begin{array}{l} 1\frac{1}{4} \\ 3\frac{1}{3} \\ 7\frac{1}{2} \\ 6\frac{2}{3} \\ 8\frac{1}{3} \\ 4\frac{2}{7} \end{array}\right. \quad ;$$

$2\frac{1}{2} \times 1\frac{1}{2}$
$1\frac{1}{2} \times 1\frac{1}{3}$
$2\frac{2}{3} \times 1\frac{1}{8}$
$3\frac{1}{3} \times 1\frac{1}{5}$
$3\frac{1}{3} \times 3\frac{1}{3}$
$2\frac{1}{2} \times 3\frac{1}{2}$

;

$4\frac{1}{3} \times 3$
$3\frac{3}{4} \times 4$
$5\frac{1}{3} \times 6$
$2\frac{2}{3} \times 3$
$1\frac{1}{2} \times 2$
$1\frac{1}{4} \times 8$

72 Written exercise:

$5\frac{5}{6} \times 8\frac{4}{7}$
$7\frac{1}{5} \times 4\frac{3}{8}$
$4\frac{1}{12} \times 5\frac{1}{7}$
$3\frac{1}{8} \times 2\frac{2}{5}$

;

$\frac{4}{7}$ of $12\frac{3}{4}$
$\frac{3}{5}$ of $5\frac{5}{9}$
$\frac{2}{5}$ of $8\frac{3}{4}$
$\frac{3}{11}$ of $8\frac{4}{5}$

;

$3\frac{1}{3} \times 2\frac{7}{8} \times 6\frac{2}{5}$
$7\frac{6}{7} \times 2\frac{3}{11} \times 4\frac{1}{5}$
$8\frac{1}{6} \times 1\frac{1}{11} \times 3\frac{1}{7}$
$4\frac{1}{5} \times 4\frac{2}{7} \times 1\frac{3}{5}$

Rule: To find the product of two expressions, either or both being fractional, first, change any whole or mixed expressions to fractions, next, cancel, if possible; then, multiply these numerators for the numerator of the product, and these denominators for the denominator of the product.

73 Written exercise:

(1) What is the cost of 25 pounds of sugar at $6\frac{1}{2}$ cents a pound?

(2) What is the cost of $\frac{3}{4}$ of a yard of ribbon at 75 cents a yard? If I pay for it with a $1 note, how much change should I receive?

(3) How many square feet of surface in a blackboard $4\frac{1}{2}$ feet by $3\frac{2}{3}$ feet?

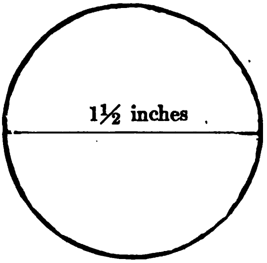
(4) $3\frac{7}{8} \times 4\frac{4}{5}$ is how much more than $2\frac{3}{8} \times 6\frac{2}{5}$?

(5) How many days in $2\frac{3}{5}$ years of 365 days each?

(6) Multiply the sum of $3\frac{5}{8}$, $2\frac{3}{4}$, and $4\frac{1}{2}$ by $10\frac{2}{3}$.

(7) What must I pay the grocer for 6 pounds of sugar at $6\frac{1}{2}$ cents, $1\frac{1}{2}$ pounds of coffee at 38 cents, and $\frac{1}{4}$ pound of tea at 60 cents?

(8) How high will a pile of 100 boards be if each board is $\frac{5}{16}$ of an inch thick (or high)?



(9) A man having a farm of 400 acres sold $\frac{4}{7}$ of it. How many acres did he sell? How many did he keep?

(10) If the line around the circle is $3\frac{1}{7}$ times $1\frac{1}{2}$ inches as long as the line through the centre, how long is the line around it?

LESSON XI

Review of Tables

74 What have you measured with the inch? The foot? How many inches make a foot? What things are bought by the yard? How many feet are in a yard? How many inches? The length of city lots, streets, roads, and farms is found with a larger measure, the *rod*, which contains 5½ yards. How many feet in a rod? 320 rods make a mile. How many rods in ½ mile? In ¼ mile?

Memorize: Table of Long Measure.

12 inches = 1 foot
3 feet = 1 yard
5½ yards = 1 rod
320 rods = 1 mile
5280 feet = 1 mile

75 Oral:

How many inches in
5 ft.?
4 ft.?
1 yd.?
½ yd.?
¼ yd.?
2½ ft.?
1¼ ft.?

How many feet in
5 yd.?
3 yd.?
60 in.?
72 in.?
1 rd.?
3 rd.?

How many yards in
15 ft.?
24 ft.?
72 in.?
2 rd.?
4 rd.?
8 rd.?

76 Written exercise:

(1) If $2\frac{1}{2}$ yards of ribbon is cut into 15-inch lengths, how many pieces will there be?
(2) It is $\frac{3}{4}$ of a mile from the house to the school. How many feet is it?
(3) A street is $\frac{1}{4}$ of a mile long. How many rods of curbing are needed for both sides?
(4) A square lot is 5 rods on a side. How many feet of fencing are needed to enclose it?
(5) What will a mile of curbing cost at $0.15 a foot?

By what measures is milk sold? How many gills make one pint? How many pints make one quart?

1 gallon = how many quarts = how many pints = how many gills?

1 gill
1 pint = what part of a gallon?
1 quart

How many pints in
2 gal.?
3 qt.?
$2\frac{1}{2}$ qt.?
$\frac{1}{2}$ gal.?
$\frac{3}{4}$ gal.?

How many quarts in
3 gal.?
12 gal.?
10 pt.?
16 pt.?
$\frac{1}{2}$ gal.?

Memorize: Table of Liquid Measure.

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

77 Written exercise:

(1) A dealer pays $0.85 for a gallon of cream. If he sells it at $0.15 a pint, how much does he gain?

(2) How many gill bottles can be filled from 5 gallons of cream?

(3) A dealer puts 20 gallons of milk into quart bottles. If he sells $\frac{4}{5}$ of them, how many quarts does he sell?

(4) How many gallons of milk at 9 cents a quart bring in $2.88 (288 cents)?

(5) How much does our milk cost in 4 weeks, if we get a pint a day and pay 5 cents a pint?

Name some things that are bought by bushels, pecks, and quarts.

1 bushel = how many pecks = how many quarts = how many pints?

1 quart = what part of a peck = what part of a bushel?

How many pecks in	2 bu.?	How many quarts in	2 bu.?
	$3\frac{1}{2}$ bu.?		$\frac{1}{2}$ bu.?
	$\frac{3}{4}$ bu.?		3 pk.?
	16 qt.?		$\frac{1}{2}$ pk.?
	40 qt.?		8 pt.?
	$\frac{1}{2}$ bu.?		10 pt.?

Memorize: Table of Dry Measure.

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel

78 Written exercise:

(1) A farmer sold 2 bushels of cherries at 10 cents a quart. What did he receive for them?

(2) 10 bushels of peaches were sold by the quarter-peck. How many quarter-pecks were sold?

(3) What did 12 bushels of potatoes bring at 25 cents a peck?

(4) George picked 64 quarts of cherries. How many bushel baskets did they fill?

(5) A barrel of apples holds 3 bushels. How many pecks does it hold?

How many hours in
1 da.?
$\frac{1}{2}$ da.?
$\frac{3}{4}$ da.?
3 da.?
$1\frac{1}{2}$ da.?

How many minutes in
1 hr.?
$\frac{1}{2}$ hr.?
3 hr.?
$1\frac{1}{2}$ hr.?
$1\frac{1}{4}$ hr.?

How many days in
3 wk.?
$\frac{1}{7}$ wk.?
$\frac{1}{2}$ mo.?
3 mo.?
1 yr.?

Memorize: Table of Time.

60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
30 days = 1 month
12 months = 1 year
365 days = 1 year

79 Written exercise:

(1) If an express train goes a mile a minute, how far does it go in an hour?

(2) If a boy reads 2 chapters every day, how many chapters does he read in 6 weeks?

(3) If a man earns $0.60 an hour, what does he earn in an eight-hour day?

(4) If a boy saves 2 cents a day, how much will he save in a year?

(5) If a man works 8 hours a day every day but Sunday, how many hours does he work in 4 weeks?

LESSON XII

Review of U. S. Money. Bills and Accounts

80 Table of U. S. Money.

10 cents make one dime.
10 dimes make one dollar.
10 dollars make one eagle.

Denominations used in U. S. Money.

material	name		value
copper	cent	=	1 cent
nickel	nickel	=	5 cents
silver	dime	=	10 cents
	quarter	=	25 cents
	half-dollar	=	50 cents
	dollar	=	100 cents
gold	eagle	=	10 dollars
	half-eagle	=	5 dollars
	quarter-eagle	=	2½ dollars
paper	bills of many values		

Caution.—Remember, when writing amounts of U. S. money to add or subtract, to place the cents exactly under cents, the point which separates dollars from cents exactly under a point, dollars under dollars.

Written exercise:

(1) Add $2.50, $7.95, $18.10, $16.95, $140.73, and $10.

(2) $49.10 + $19.75 + $17.95 + $75.38 + $10.75 = what sum?

(3) Add \$4.50, \$0.95, \$0.65, \$0.27, and \$0.19. Take the sum from \$10.

(4) From \$21.74 + \$19.38 take \$16.05 − \$8.69.

(5) Add fourteen dollars, thirteen cents; seventy dollars, nine cents; fifty-six cents; sixty cents; and twelve cents.

(6) Add a dime, a quarter, a half-dollar, seven nickels, and nineteen cents.

(7) From \$135.07 take \$38.98. Then add the remainder to \$17.14.

(8) How much change would you have from \$5, if you should pay for 16 pounds of sugar at \$0.06 a pound, and 3 gallons of molasses at \$0.38 a gallon?

(9) How much would you have to pay for 3 pecks of potatoes at \$0.28 a peck, 1 peck of apples at \$0.35 a peck, and 3 dozen bananas at \$0.15 a dozen?

(10) How much more is \$9250.65 than \$8094.72?

(11) How much more is the sum of 11 quarters, 5 half-dollars, 16 dimes, 17 nickels, and 19 cents, than \$4.85?

(12) Tom and Nan opened their banks; Tom had 19 nickels, 3 dimes, and 27 cents in his; Nan had 49 cents, 7 nickels, and 1 dime in hers. Who had more and how much?

(13) How much less than \$5 are 69 nickels?

(14) How much must be added to \$3.78 to be able to pay for 7 books at \$0.75 apiece?

(15) The conductor on the trolley car rang up 69 fares. How much money did he take on that trip if 47 of the passengers were grown folks?

81 A BILL is a form used by a person who has sold goods to another person, or done work for him, to tell him how much the goods or the work has cost. The person who owes the money is called the DEBTOR. The form is like this:

Baltimore, Md., June 15, 1915.

Mrs. W. H. Brown

To Henry M. Davidson, Dr.
Groceries and General Supplies.
716 West Street.

June.	1	5 lbs. granulated sugar,	@ $.06		30		
	5	2 lbs. coffee,	@ .35		70		
	10	2 boxes puffed wheat,	@ .12		24	1	24

Who sold these goods? Who bought them?

How many items are there? When were they bought? When was the bill made?

When the bill is paid by the debtor, the words "Received payment" and the creditor's name and date are placed in the lower right-hand corner. This is called *receipting the bill.* (The creditor is the man to whom the money is owed.) Who should receipt this bill?

(1) Geo. T. Ellis, fruit dealer, sold to Mrs. James T. Morris, March 3, 1915, 3 dozen oranges at $0.25 a dozen; March 5, ½ dozen grape fruit at $0.08 apiece; and March 8, 3 pounds of figs at $0.15 a pound. Make out this bill, dating it March 16, and receipt it 10 days later.

(2) Make out a bill to James Field for $18 worth of carpentering work done to his barn by Thomas Daly. Receipt it 5 days after the bill is supposed to be made.

(3) Make Dr. E. S. Stevens' bill of $25.10 for dentistry done for the children of Mrs. Geo. Mason.

(4) Make out a bill of four items that your grocer might send your mother. Receipt it.

(5) Make a bill that you might send your neighbor for cutting the grass on her lawn three times; or for cleaning the snow from her sidewalk three times.

Note.—The teacher should give more practice in *Bills and Accounts* if she thinks it advisable.

82 Review of fundamental processes in integers, and in fractions.

I. (1)	(2)	(3)	(4)	(5)
+6854	+684	+ 8176	+98	+80165
9217	965	897	89	2178
8614	987	87	79	98
2198	149	689	97	149
7689	867	5432	65	1568
4136	432	869	56	98764
1258	986	17439	49	2186
	745	8765	94	149
			125	1909

Write the sums in words.

II. Take	29876	from 1000000	Write the remainders in words.
	19839	901060	
	4087	705111	
	124369	2012345	
	14976	709861	
	50865	2107003	
	98143	901003	
	121694	687592	Check each.
	7098	150123	
	98	187013	

III. Multiplicands: 986, 409, 298, 497, 217, 912.
Multipliers: 986, 864, 709, 219, 765, 846.
Find products. Check each. KNOW that each is correct.

IV. Dividends:	7913	Divisors:	896	Check each in one of the two ways.
	901605		109	
	21698		2186	
	914360		905	
	21687		863	
	2198		1234	
	43876		869	
	218761		75	
	14260		475	
	987614		1098	

V. Find by division H. C. F. of:
75, 125, 450; 36, 48, 60; 45, 90, 6;
15, 60, 35; 18, 24, 42; 24, 32, 40.

VI. Add: $4\frac{1}{2}$, $6\frac{1}{3}$, $9\frac{4}{5}$, $10\frac{9}{10}$
$6\frac{1}{4}$, $7\frac{3}{8}$, $7\frac{11}{20}$, $2\frac{1}{2}$ Add vertically.
$7\frac{5}{8}$, $9\frac{1}{2}$, $12\frac{37}{40}$, $19\frac{17}{20}$ Add horizontally.
$13\frac{5}{16}$, $7\frac{15}{16}$, $8\frac{1}{2}$, $9\frac{7}{8}$
$8\frac{11}{32}$, $12\frac{3}{4}$, $17\frac{5}{8}$, $19\frac{1}{2}$

VII. Minuends: $98\frac{1}{2}$, $104\frac{2}{5}$, 101, $99\frac{7}{15}$, $291\frac{1}{3}$.
Subtrahends: $16\frac{7}{9}$, $87\frac{19}{30}$, $36\frac{5}{17}$, $14\frac{9}{20}$, $49\frac{19}{20}$.
Find remainders.

From $117\frac{1}{3}$ take $19\frac{4}{5}$; from the remainder, $29\frac{5}{6}$; from this remainder, $43\frac{11}{12}$.

From $907\frac{4}{9}$ take $193\frac{23}{45}$; from the remainder, $209\frac{2}{3}$; from this remainder, $16\frac{1}{2}$.

VIII.

$$\left.\begin{array}{r}4\\9\\8\\7\\6\\5\\4\\15\end{array}\right| \quad \left|\begin{array}{l}2\frac{1}{3}\\9\frac{1}{2}\\6\frac{4}{9}\\9\frac{2}{3}\\6\frac{1}{5}\\7\frac{1}{3}\\9\frac{1}{4}\\2\frac{1}{6}\end{array}\right. = ? \qquad \left.\begin{array}{r}2\frac{1}{5}\\1\frac{2}{3}\\7\frac{4}{5}\\8\frac{7}{8}\\9\frac{5}{6}\\4\frac{2}{7}\\6\frac{3}{4}\\7\frac{1}{2}\end{array}\right| \times \left|\begin{array}{l}2\\7\\8\\9\\3\\4\\2\\6\end{array}\right. = ? \qquad \left.\begin{array}{c}\frac{1}{7}\\\frac{1}{8}\\\frac{7}{9}\\\frac{5}{6}\\\frac{11}{12}\\\frac{7}{16}\\\frac{2}{3}\end{array}\right| \times \left|\begin{array}{c}\frac{7}{15}\\\frac{16}{17}\\\frac{27}{35}\\\frac{24}{20}\\\frac{25}{33}\\\frac{24}{21}\\\frac{9}{50}\end{array}\right. = ?$$

$2\frac{1}{6} \times 7\frac{1}{4} = ?$
$5\frac{3}{5} \times 3\frac{5}{6} = ?$
$7\frac{1}{2} \times 2\frac{1}{2} = ?$
$9\frac{1}{3} \times \frac{6}{7} = ?$
$1\frac{5}{12} \times \frac{19}{34} = ?$
$6\frac{1}{4} \times \frac{27}{50} = ?$
$2\frac{1}{3} \times \frac{15}{28} = ?$

LESSON XIII

Division of Fractions

83 $1 \div 3$
$\frac{1}{3}$

These are different ways of indicating that 1 is divided by 3.

$1 \div 3$
$\frac{1}{3}$ of 1
$1 \times \frac{1}{3}$

1 divided by 3 is $\frac{1}{3}$ of 1.
Therefore dividing by 3 is
multiplying by $\frac{1}{3}$.

Now, divide 1 by $\frac{3}{4}$. Since the divisor $\frac{3}{4}$ is $\frac{1}{4}$ of the divisor 3, the quotient will be 4 times $\frac{1}{3}$ of 1, for you know that when the divisor is made $\frac{1}{4}$ as large, the quotient becomes 4 times as great. $(24 \div 8 = 3)$ $(24 \div 2 = 12)$

$1 \div \frac{3}{4} =$
$1 \times \frac{4}{3}$

Therefore, dividing 1 by $\frac{3}{4}$ is multiplying 1 by $\frac{4}{3}$. Hence, for the same reason, dividing by any fraction is multiplying by that fraction inverted.

Oral exercise:

$1 \div 3$
$1 \div 5$
$1 \div 7$
$1 \div 9$
$1 \div 10$

$2 \div 3$
$3 \div 4$
$5 \div 8$
$6 \div 7$
$9 \div 12$

$1 \div \frac{1}{3}$
$1 \div \frac{1}{4}$
$1 \div \frac{1}{5}$
$1 \div \frac{1}{6}$
$1 \div \frac{1}{8}$

$1 \div \frac{2}{3}$
$1 \div \frac{4}{5}$
$1 \div \frac{7}{8}$
$1 \div \frac{5}{9}$
$1 \div \frac{7}{10}$

$2 \div \frac{1}{3}$
$4 \div \frac{5}{6}$
$10 \div \frac{3}{4}$
$8 \div \frac{2}{3}$
$3 \div \frac{3}{4}$

84 Written exercise:

$$18 \div \frac{3}{5} = \frac{\overset{6}{\cancel{18}}}{1} \times \frac{5}{\underset{1}{\cancel{3}}} = \frac{6\times 5}{1} = 30 = \text{Quotient.}$$

$$30 \times \frac{3}{5} = \frac{\overset{6}{\cancel{30}}\times 3}{1\times \underset{1}{\cancel{5}}} = \frac{6\times 3}{1} = 18 = \text{Proof-check.}$$

$$\left.\begin{array}{r}4\\12\\16\\20\\24\\8\end{array}\right| \div \frac{4}{9} \quad ; \quad \left.\begin{array}{r}6\\12\\18\\15\\30\\27\end{array}\right| \div \frac{3}{7} \quad ; \quad \left.\begin{array}{r}10\\25\\15\\30\\40\\35\end{array}\right| \div \frac{5}{8} \quad ; \quad 100 \div \left|\begin{array}{c}\frac{1}{2}\\\frac{3}{4}\\\frac{4}{5}\\\frac{8}{9}\end{array}\right. \quad ; \quad 144 \div \left|\begin{array}{c}\frac{9}{10}\\\frac{8}{9}\\\frac{6}{7}\\\frac{4}{9}\end{array}\right.$$

85 Oral:

$\frac{1}{2} \div \frac{1}{4}$
$\frac{1}{3} \div \frac{1}{12}$
$\frac{1}{6} \div \frac{1}{18}$
$\frac{1}{8} \div \frac{1}{40}$
$\frac{1}{10} \div \frac{1}{50}$

$\frac{2}{3} \div \frac{4}{3}$
$\frac{8}{9} \div \frac{2}{9}$
$\frac{7}{10} \div \frac{7}{20}$
$\frac{2}{5} \div \frac{2}{15}$
$\frac{7}{8} \div \frac{7}{8}$

$\frac{3}{4} \div \frac{1}{2}$
$\frac{2}{3} \div \frac{1}{3}$
$\frac{3}{5} \div \frac{1}{10}$
$\frac{5}{6} \div \frac{1}{6}$
$\frac{9}{10} \div \frac{3}{10}$

$\frac{4}{9} \div 2$
$\frac{3}{10} \div 3$
$\frac{5}{8} \div 5$
$\frac{8}{15} \div 4$
$\frac{9}{10} \div 3$

$\frac{1}{2} \div 5$
$\frac{1}{3} \div 4$
$\frac{1}{5} \div 5$
$\frac{1}{10} \div 3$
$\frac{1}{12} \div 2$

86 Written exercise:

$$\frac{1}{12} \div \frac{8}{9} = \frac{1}{\underset{4}{\cancel{12}}} \times \frac{\overset{3}{\cancel{9}}}{8} = \frac{1\times 3}{4\times 8} = \frac{3}{32} = \text{Quotient}$$

$$\frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{32}}} \times \frac{\overset{1}{\cancel{8}}}{\underset{3}{\cancel{9}}} = \frac{1\times 1}{4\times 3} = \frac{1}{12} = \text{Proof-check}$$

$\frac{4}{5} \div \frac{6}{7} \qquad \frac{17}{20} \div \frac{4}{15} \qquad \frac{1}{12} \div \frac{8}{9} \qquad \frac{1}{9} \div \frac{6}{7} \qquad \frac{87}{100} \div \frac{5}{6}$
$\frac{14}{15} \div \frac{1}{3} \qquad \frac{2}{3} \div \frac{8}{9} \qquad \frac{5}{8} \div \frac{5}{12} \qquad \frac{3}{7} \div \frac{7}{9} \qquad \frac{75}{100} \div \frac{3}{4}$
$\frac{19}{24} \div \frac{3}{16} \qquad \frac{13}{24} \div \frac{13}{30} \qquad \frac{9}{10} \div \frac{9}{10} \qquad \frac{14}{15} \div \frac{21}{25} \qquad \frac{7}{11} \div \frac{9}{22}$
$\frac{7}{11} \div \frac{9}{22} \qquad \frac{6}{7} \div \frac{7}{8} \qquad \frac{25}{32} \div \frac{5}{16} \qquad \frac{33}{50} \div \frac{11}{40} \qquad \frac{21}{25} \div \frac{7}{20}$
$\frac{14}{25} \div \frac{14}{50} \qquad \frac{5}{8} \div \frac{25}{36} \qquad \frac{20}{21} \div \frac{10}{63} \qquad \frac{57}{100} \div \frac{19}{25} \qquad \frac{13}{24} \div \frac{13}{48}$

When the dividend or divisor is a mixed number, change it to a fraction.

$12\frac{1}{2} \div 2\frac{1}{2} = \frac{25}{2} \div \frac{5}{2} = \frac{\overset{5}{\cancel{25}}}{\cancel{2}} \times \frac{\overset{1}{\cancel{2}}}{\cancel{5}} = \frac{5 \times 1}{1 \times 1} = 5 =$ Quotient.

$5 \times 2\frac{1}{2} = \frac{5}{1} \times \frac{5}{2} = \frac{5 \times 5}{1 \times 2} = \frac{25}{2} = 12\frac{1}{2} =$ Proof-check.

87 Written exercise:

$12\frac{1}{2} \div 6\frac{1}{4} \qquad 5\frac{1}{2} \div 3\frac{2}{3} \qquad 6\frac{7}{8} \div 6\frac{3}{5} \qquad 6\frac{2}{3} \div 4 \qquad 6\frac{1}{9} \div \frac{11}{15}$
$37\frac{1}{2} \div 3\frac{3}{4} \qquad 7\frac{2}{3} \div 4\frac{2}{3} \qquad 7\frac{3}{4} \div 5\frac{1}{6} \qquad 3\frac{3}{8} \div 9 \qquad 5\frac{1}{10} \div \frac{3}{5}$
$66\frac{2}{3} \div 16\frac{2}{3} \qquad 4\frac{8}{9} \div 2\frac{4}{9} \qquad 8\frac{4}{5} \div 6\frac{1}{9} \qquad 8\frac{3}{4} \div \frac{7}{8} \qquad 66\frac{2}{3} \div \frac{2}{3}$
$5\frac{1}{3} \div 2\frac{2}{3} \qquad 62\frac{1}{2} \div 4\frac{1}{6} \qquad 5\frac{7}{10} \div 3\frac{4}{5} \qquad 37\frac{1}{2} \div 5 \qquad 62\frac{1}{2} \div \frac{5}{8}$
$25\frac{1}{2} \div 2\frac{5}{6} \qquad 30\frac{1}{4} \div 4\frac{7}{12} \qquad 5\frac{2}{5} \div 2\frac{7}{10} \qquad 30\frac{1}{4} \div 11 \qquad 100 \div 12\frac{1}{2}$

88 Rule: To divide one expression by another, either or both being fractional, first, change any whole or mixed expressions to fractions; next, invert the DIVISOR; then, multiply.

Written exercise:

(1) How many pieces of ribbon $\frac{5}{8}$ of a yard long can be sold from 20 yards?

(2) How many steps of 2½ feet each would a boy take in walking 50 yards?

(3) If 110⅖ bushels of wheat are grown on 4⅘ acres of land, how many bushels are grown on each acre?

(4) If a man receives $22⅘ for 6 days' work, what does he earn a day?

(5) How many 2½ bushel baskets will be needed to hold 62½ bushels of peaches?

(6) If $226½ is paid for 6 acres of land, what is the price per acre?

(7) How many kite strings 75⅓ yards long can be gotten from 226 yards of cord?

(8) How many oranges selling at 2 for 5 cents can be gotten for 25 cents?

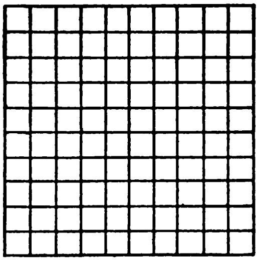
(9) A piece of goods containing 6¼ yards cost a dollar. How many cents did each yard cost?

(10) How many sheets 2½ yards long can be made from 40 yards of muslin?

LESSON XIV

Introduction to Decimals

89 This square is divided into 10 equal rows, and each row into 10 equal parts. How many of these small parts in the big square? Each one is therefore $\frac{1}{100}$ of the big square. Each is what % of the big square? One row is what %? Three rows? Seven rows? Five small squares are what %?



1% of a thing is what part of the thing? 1% = $\frac{1}{100}$.
10% of a thing is what part of the thing? 10% = $\frac{10}{100}$.

Another way to write $\frac{1}{100}$ is .01.
Another way to write $\frac{1}{10}$ is .1.

This new kind of fraction is called decimal fraction, because its denominator, which is not written, is 10 or some multiple of 10. (*Decimal* comes from *decem*, which means *ten*.) The part which is written is the numerator; the denominator is 1 with as many ciphers as the numerator is places away from the decimal point.

Thus: $\frac{1}{10} = .1$
$\frac{2}{10} = .2$
$\frac{3}{10} = .3$
. .
. .
. .
. .
. .
$\frac{9}{10} = .9$
$\frac{10}{10} = 1.0$

$\frac{1}{100} = .01$
$\frac{2}{100} = .02$
$\frac{3}{100} = .03$
. .
. .
. .
. .
$\frac{90}{100} = .90$
$\frac{90}{100} = .99$
$\frac{100}{100} = 1.00$

Read: .9, .8, .7, .6, .4, .2, .8, .5, .1, .17, .18, .27, .64, .12, .09, .87, .91, .95.

2.1 is read two and one-tenth.

17.07 is read seventeen and seven-hundredths.

NOTE.—Use the word AND between the integral and the fractional parts of a mixed expression.

Read:
1.5
10.7
11.8
101.2
16.9
.73

1.21
1.25
17.13
8.09
19.45
16.09

.7
.17
.9
.09
.73
.16

2.01
3.09
3.17
4.9
16.5
16.05

Written exercise: Express each of the above decimal fractions as common fractions.

Write as decimal fractions: $\frac{1}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, $\frac{9}{10}$, $\frac{6}{10}$, $\frac{5}{10}$, $\frac{9}{100}$, $\frac{19}{100}$, $\frac{6}{100}$, $\frac{64}{100}$, $\frac{13}{100}$, $\frac{14}{100}$, $\frac{87}{100}$, $\frac{50}{100}$, $\frac{61}{100}$, $\frac{98}{100}$, $\frac{12}{100}$, $\frac{8}{100}$, $\frac{7}{100}$, $\frac{5}{100}$, $\frac{11}{100}$, $\frac{1}{100}$, $\frac{18}{100}$.

90 It is just as easy to add or subtract decimal fractions as it is to add or subtract integers.

CAUTION.—Write the numbers in even columns, decimal point under decimal point, tenths under tenths, and hundredths under hundredths.

$$\begin{array}{rl} \left.\begin{array}{r}+.9\\ .7\\ .6\\ .4\end{array}\right| & =\text{addends}\\ \hline 2.6 & =\text{sum}\end{array} \qquad \begin{array}{rl} \left.\begin{array}{r}+.91\\ .16\\ .25\\ .43\end{array}\right| & =\text{addends}\\ \hline 1.75 & =\text{sum}\end{array}$$

$$\begin{array}{rl} .7 & =\text{minuend.}\\ -\ .3 & =\text{subtrahend.}\\ \hline .4 & =\text{remainder.}\end{array}$$

$$\begin{array}{rl} 9.6 & =\text{minuend.}\\ -2.7 & =\text{subtrahend.}\\ \hline 6\ 9 & =\text{remainder.}\end{array}$$

91 Written exercise:

+9.4	+3.2	+7.3	+2.1	+7.2	+9.9
3.2	7.5	2.9	7.9	9.8	8.8
6.9	8.1	8.7	8.7	9.6	7.7
7.4	2.7	1.4	6.9	7.3	6.6
2.7	3.6	6.9	4.3	4.5	5.5
8.7	4.5	8.7	8.7	7.2	4.4

Write the names of the addends and sums.

Set down carefully, and add the horizontal lines.

+18.63	+17.63	+19.09	+13.97	+ 7.08	+16.74
19.75	19.76	18.08	97.63	6.09	71.36
7.43	19.37	17.71	14.13	7.12	93.73
6.08	14.09	36.75	13.19	8.15	31.49
7.14	15.08	14.07	19.63	7.96	16.09
2.09	17.73	15.06	17.07	18.14	27.36

Add the vertical columns.

Write the names of the sums.

Add: .9, .16, .15, .93, .7, .6, .63.
.43, .7, .65, .67, .71, .9, .2.
2.1, 7.5, 6.16, 9.63, 7.5, 4.9.
7.8, 2.08, 7.9, 1.09, 8.5, 7.05.
9.5, 9.05, 7.1, 7.01, 9.53, 9.78.

92	.9	4.7	7.5	6.3	5.1	7.2
	−.6	−2.9	−2.9	−1.9	−2.8	−1.7

	1.2	Take 9.2	from 15.1
	3.9	7.9	16.2
9.7 −	7.8	8.7	17.3
	1.2	6.9	10.3
	2.7	8.1	9.6
	5.8 = ?	4.7	12.2
	1.9	4.9	13.1
	2.3	3.6	10.7
	4.5	4.3	15.2
	7.5	5.5	19.1

	4.09	From 100.75	take 36.49
	5.16	98.63	54.75
	7.37	75.01	64.01
	5.95	64.37	49.57
20.15 −	1.23	73.64	18.43
	17.68	115.07	29.64
	19.43	103.98	58.09
	12.09	97.73	14.08
	7.19	84.64	15.87
	8.74	73.89	16.94

LESSON XV

General Review

93 I. 1. Add, subtract, multiply, and divide 982 and 75; then add the four results. Write the final result in words.

2. Add, subtract, and divide 750 and 25. Add the three results. Multiply this sum by 19. Write the final result in words.

3. Add 1943, 8764, 9874, 9217, 8179, 6493, 8196. Multiply the sum by 67. Write the name of the product.

4. Subtract (86 + 94 + 98 + 65 + 195) from (129 + 687 + 436 + 986 + 894).

5. (687 + 986 + 753 + 986) × (687 − 289) = what product?

II.
9876 × 73
2168 × 165
9132 × 84
2168 × 29
7605 × 75
= what products?

Arrange in order of value. Write the name of the smallest.

How much more is the greatest than the smallest?

III.
690053
421768
178921
112300
112345
÷
125
336
249
108
963
= ?

Check each problem in one of the two ways you know.

IV. 694300 ÷ 425 | Subtract the lesser quotient from
123456 ÷ 649 | the greater. Multiply the remainder by $\frac{4}{5}$ of 35.

V. (1) The items of a bill were 15 pounds of sugar at $0.06; 3 pounds of coffee at $0.28; $\frac{1}{2}$ pound of baking powder at $0.50 a pound; $1\frac{1}{2}$ pecks of potatoes at $0.30 a peck. What was the amount of the bill? How much change would a person get from $10?

(2) Make out the bill for the above to your mother. Receipt it.

(3) Add $9.65, $17.39, $4.25, $0.67, $0.29, $2.19. Take the sum from $50.

(4) Which costs more: 15 yards of material at $0.45, or 23 yards at $0.25? How much more?

(5) Five of the amounts in an account were $967.31, $439,17, $2169.24, $29.68, $143.17. What would they add to?

VI.

(1)	(2)	(3)	(4)	(5)
+19.27	+27.90	+29.05	+ 9.1	+17.2
16.43	17.09	27.09	19.2	96.4
14.96	29.63	29.36	34.3	16.4
21.07	49.17	43.29	49.2	29.8
68.94	27.45	14.26	36.9	74.3
23.16	36.73	47.31	117.3	75.9

Arrange the sums in order of value. Add them.

Write the name of their sum.

Take the smallest from the largest. Write the name of the remainder.

Take the third one in value from the largest one.

VII. Write the table of liquid measure. Of dry measure. Long measure. Square measure. Time measure.

(1) How many feet in 1 mile? In 9 miles?

(2) How many feet around an oblong 3 miles by 5 miles?

(3) How many yards around the same oblong?

(4) One city lot is 25 feet wide by 75 feet long; another is 50 feet by 34 feet. Which has the greater perimeter? Which has the greater number of square feet?

(5) How much more is one worth than the other if one square foot is worth $1.25?

(6) How many months old are you?

(7) A boy earned $0.05 every day for three years. How much did he earn?

(8) A newsboy sold 60 papers every week-day for 10 weeks. How much did he get for them at 1 cent each?

(9) He had to pay $\frac{2}{3}$ of that amount for the papers. How much did he gain on them?

(10) What will 25 pecks of berries cost at $0.08 a quart?

(11) What will 25 gallons of milk cost at \$0.09 a quart?

VIII. (1) Name the prime numbers up to 100.

(2) Name the prime factors of 18, 96, 100, 75, 27, 36, 48, 60, 72, 15.

(3) Name the common prime factors of 60, 75; 48, 72; 15, 45; 27, 63; 24, 36; 35, 70

(4) Find the H. C. F. in two ways of the numbers in No. 3.

(5) Find the L. C. M.* of the numbers in No. 3.

(6) Change to equivalent fractions having lowest common denominator:
$\frac{2}{3}$, $\frac{3}{4}$; $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$; $\frac{9}{10}$, $\frac{4}{5}$, $\frac{2}{3}$; $\frac{7}{8}$, $\frac{3}{4}$, $\frac{2}{3}$; $\frac{1}{7}$, $\frac{1}{14}$, $\frac{20}{21}$; $\frac{1}{5}$, $\frac{9}{10}$, $\frac{4}{15}$; $\frac{6}{11}$, $\frac{3}{22}$, $\frac{19}{33}$; $\frac{19}{20}$, $\frac{7}{10}$, $\frac{7}{8}$, $\frac{11}{12}$, $\frac{11}{15}$.

(7) How many dollars in fifty-nine quarters? In 300 half-dollars?

(8) How many pecks in $\frac{311}{8}$ of a peck? How many gallons in $\frac{311}{4}$ of a gallon?

(9) Change to mixed expressions: $\frac{95}{12}$, $\frac{69}{5}$, $\frac{416}{25}$, $\frac{372}{15}$, $\frac{96}{7}$, $\frac{85}{3}$, $\frac{415}{50}$.

(10) Change to improper fractions: $9\frac{1}{7}$, $16\frac{4}{15}$, $25\frac{9}{16}$, $18\frac{3}{20}$, $75\frac{9}{14}$, $17\frac{1}{9}$, $9\frac{53}{60}$.

(11) Arrange in order of value $\frac{4}{5}$, $\frac{3}{4}$, $\frac{11}{15}$, $\frac{5}{6}$, $\frac{13}{20}$, $\frac{43}{60}$.

---

*The term *lowest common multiple* is sometimes called *least common dividend*.

IX. Add:

$9\frac{1}{2}, 6\frac{1}{4}, 7\frac{5}{16}, 3\frac{5}{8}, 9\frac{11}{32}$
$2\frac{1}{7}, 3\frac{1}{2}, 8\frac{9}{14}, 9\frac{20}{21}, 7\frac{11}{42}$
$2\frac{1}{5}, 6\frac{7}{10}, 9\frac{3}{20}, 11\frac{4}{5}, 7\frac{3}{4}$
$7\frac{1}{3}, 9\frac{3}{4}, 8\frac{8}{9}, 9\frac{11}{18}, 7\frac{1}{2}$
$9, 8\frac{1}{3}, 7\frac{11}{16}, 3\frac{23}{24}, 16$

Take the fifth sum from the first.

Take the third sum from the fourth.

X.

From $17\frac{2}{3}$	take	$2\frac{4}{5}$
19		$9\frac{11}{30}$
$26\frac{4}{15}$		$7\frac{49}{60}$
$17\frac{11}{30}$		$8\frac{17}{20}$
$16\frac{9}{12}$		$2\frac{3}{4}$
$17\frac{1}{2}$		$7\frac{19}{20}$
$20\frac{3}{4}$		$3\frac{29}{30}$
$19\frac{1}{12}$		$7\frac{5}{6}$
25		$8\frac{2}{3}$
$30\frac{7}{20}$		16

Add the first five subtrahends; then the first five minuends; find the difference between their sums.

Find the difference between the sum of the last five minuends and the sum of the last five subtrahends.

XI.

$7\times\frac{9}{10}$	$=?$	$\frac{8}{9}$	$\times=8?$	$\frac{4}{5}$		$\frac{15}{16}$		$2\frac{1}{2}\times3\frac{1}{5}=$
$\frac{15}{17}$		$\frac{5}{16}$		$\frac{9}{12}$		$\frac{2}{3}$		$2\frac{1}{2}\times7\frac{2}{3}=$
$2\frac{1}{2}$		$2\frac{1}{8}$		$\frac{16}{20}$		$\frac{7}{8}$		$7\frac{1}{4}\times1\frac{1}{29}=$
$9\frac{1}{6}$		$7\frac{5}{16}$		$\frac{8}{15}$		$\frac{30}{32}$		$1\frac{1}{7}\times3\frac{1}{16}=$
$8\frac{4}{7}$		$9\frac{5}{24}$		$\frac{35}{36}$		$\frac{27}{28}$		$4\frac{1}{2}\times3\frac{5}{18}=$
$9\frac{11}{14}$		$\frac{11}{32}$		$\frac{7}{8}$	$\times$	$\frac{4}{5}$	$=?$	$1\frac{2}{3}\times1\frac{4}{5}=$
$9\frac{1}{21}$		$\frac{23}{24}$		$\frac{7}{15}$		$\frac{20}{21}$		$1\frac{1}{6}\times3\frac{1}{7}=$
$\frac{11}{35}$		$2\frac{1}{16}$		$\frac{18}{25}$		$\frac{25}{27}$		$2\frac{3}{5}\times2\frac{1}{13}=$
$\frac{16}{28}$		$8\frac{3}{4}$		$\frac{1}{2}$		$\frac{1}{3}$		$1\frac{1}{5}\times5\frac{5}{6}=$
$7\frac{1}{7}$		$7\frac{1}{2}$		$\frac{9}{16}$		$\frac{4}{7}$		$2\frac{1}{4}\times1\frac{8}{9}=$

(1) What will be the cost of $9\frac{1}{2}$ yards of lace at 25 cents a yard?

(2) What will be the cost of $4\frac{1}{2}$ acres of land at $125 an acre?

(3) One square is $36\frac{1}{4}$ feet in perimeter. Another one is only $\frac{1}{5}$ as great in perimeter. How far around the smaller square? What is one of its sides?

(4) It is $3\frac{1}{2}$ miles around a big field, but only $\frac{6}{7}$ as far around a smaller one. The smaller one is square. What is one of its sides?

(5) Multiply $\frac{4}{5}$ of $\frac{25}{36}$ by $7\frac{1}{2}$.

XII. $91 \div \frac{7}{8}$
$63 \div \frac{21}{22}$
$74 \div \frac{2}{3}$
$19 \div \frac{1}{2}$
$8 \div \frac{4}{5}$

$\frac{9}{10} \div 27$
$\frac{8}{9} \div 24$
$4\frac{1}{2} \div 20$
$6\frac{1}{3} \div 30$
$\frac{7}{8} \div 21$

$\frac{4}{5} \div \frac{16}{17}$
$\frac{2}{3} \div \frac{16}{27}$
$\frac{1}{2} \div \frac{15}{26}$
$\frac{9}{10} \div \frac{27}{40}$
$\frac{9}{14} \div \frac{27}{35}$

$2\frac{1}{3} \div 4\frac{1}{5}$
$4\frac{1}{6} \div 2\frac{1}{3}$
$1\frac{1}{2} \div 7\frac{3}{4}$
$2\frac{1}{3} \div 2\frac{1}{9}$
$1\frac{1}{5} \div 2\frac{1}{15}$

(1) Divide $29½ into 5 equal parts.

(2) Divide $9\frac{1}{3}$ dozen into 7 equal parts. How many in each part?

(3) Divide $16\frac{1}{2}$ by $5\frac{1}{2}$; then divide the quotient by $\frac{2}{3}$.

(4) $(\frac{1}{2} \div \frac{1}{3}) \div (\frac{1}{3} \div \frac{1}{2}) = ?$

(5) Divide 16 by $3\frac{1}{2}$. Multiply the quotient by 7.

(6) Find the difference between $(19 \div \frac{4}{5})$ and $(19 \div \frac{5}{6})$.

(7) Divide $\$17\frac{1}{2}$ into portions containing $\$\frac{5}{8}$. How many will there be?

(8) How many portions of $\$10\frac{1}{2}$ will come out of \$126?

(9) Quotient, $2\frac{1}{4}$; dividend, $33\frac{3}{4}$. Required: divisor.

(10) Find the quotient when the divisor is $3\frac{1}{5}$ and the dividend is $60\frac{4}{5}$.

Tables of Measurement

Long measure:

(1) 12 in. = 1 ft.
36 in. = 3 ft. = 1 yd.
198 in. = 16½ ft. = 5½ yds. = 1 rd.
63360 in. = 5280 ft. = 1760 yds. = 320 rds. = 1 mi.

Liquid measure:

(2) 4 gi. = 1 pt.
8 gi. = 2 pt. = 1 qt.
32 gi. = 8 pt. = 4 qt. = 1 gal.

Dry measure:

(3) 2 pt. = 1 qt.
16 pt. = 8 qt. = 1 pk.
64 pt. = 32 qt. = 4 pk. = 1 bu.

Time measure:

(4) 60 min. = 1 hr.
24 hr. = 1 da.
7 da. = 1 wk.
30 da. = 1 mo.
365 da. = 12mo. = 1 yr.
366 days = 1 leap year.

Square measure:

(5) 144 sq. in. = 1 sq. ft.
1296 sq. in. = 9 sq. ft. = 1 sq. yd.

United States money:

(6) 10 cts. = 1 dime
100 cts. = 10 dimes = 1 dollar

Counting table:

(7) 12 units = 1 doz.
144 units = 12 doz. = 1 gross